Note

Too much investment? A problem of endogenous outside options✩

David de Mezaa, Ben Lockwoodb,∗

a LSE, United Kingdom
b University of Warwick, United Kingdom

ARTICLE INFO

Article history:
Received 17 May 2006
Available online 16 December 2009

JEL classification:
C73
C78
D62

ABSTRACT

This paper shows that when agents on both sides of the market are heterogeneous, varying in their costs of investment, ex ante investments by firms and workers (or buyers and sellers more generally) may be too high when followed by stochastic matching and bargaining over quasi-rents. The overinvestment is caused by the fact that low-cost agents, by investing more, can increase the value of their outside option and thus shift rent away from high-cost investors. Numerical simulations show that overinvestment can occur given parameter values calibrated to OECD labour markets.

1. Introduction

According to Sicherman (1991), some 40% of US workers have more education than is needed for the job they do, whereas 16% are underqualified (Vaisey, 2006 reports similar figures). This finding might suggest that the private return to education is low. In fact, the opposite is the case. For example Carneiro et al. (2003) estimate that the average annual return to college for a randomly selected US citizen is 18.7%. How can these two facts be reconciled?

The standard reconciliation, following Spence (1973), is that education does not so much create human capital as signal otherwise unobservable productivity. This paper develops an alternative explanation. Since education widens employment opportunities, it increases bargaining power even in jobs for which it is unnecessary. For example, a firm may hire an available MBA graduate because finding a suitable candidate without an MBA involves delay. To make such an appointment, the pay must reflect what the MBA is worth to firms for which it does have value; i.e. the firm may have to pay the MBA candidate the value of his “outside option.” Acquiring an MBA might therefore be privately beneficial, even if that individual then takes a job for which it is unnecessary. As a result, there is a tendency to overinvestment from a social perspective, but the specific mechanism differs from the standard signalling story; overinvestment occurs because it enables the investor to capture rents, rather than being used as a signalling device.

The model developed in this paper builds on a growing literature on ex ante investments followed by stochastic matching, with the rent from the match divided by ex-post bargaining (Acemoglu, 1996, 1997; Acemoglu and Shimer, 1999; and Masters, 19981). The bargaining protocols studied in these papers allow for agents to have outside options, either in the form of a return to the search process or in switching between partners. Nevertheless, in these papers, unlike in ours, outside options do not generate overinvestment. This is because they do not bind in equilibrium, or indeed for small deviations

✩ We are grateful to three referees, the Editor, Daron Acemoglu, John Black, Bhaskar Dutta, John Hardman-Moore, Alan Manning, Chris Pissarides, Kevin Roberts and seminar participants at Universities of Birmingham, Cambridge, Kent, Vienna and Warwick, and at EUI and UCL for helpful comments.

∗ Corresponding author.

E-mail address: b.lockwood@warwick.ac.uk (B. Lockwood).

1 Masters (1998) claims that when the outside option takes the form of switching between partners, the outcome is efficient. De Meza and Lockwood (2006) show that there is an error in this argument: if switching is costless, as in Masters, there are a continuum of investment equilibria between the hold-up and efficient levels, and if there is any cost ε > 0 of switching, only the hold-up level of investment can be an equilibrium.
from equilibrium investments. So, any agent increasing their investment by a small enough increment faces the standard hold-up problem i.e. only receives half the return on their investment, implying underinvestment from a social viewpoint. However, in all these contributions, agents are assumed homogeneous. In this paper, we show that if agents are heterogeneous, differing in investment costs, outside options may become binding in equilibrium, and thus may provide incentives for overinvestment. This works as follows. We assume that a fraction of firms and workers have a low cost of investment (low-cost agents) and the complementary fraction have a high cost of investment (high-cost agents). Suppose further for simplicity that high-cost agents do not invest in equilibrium, because the cost of investment is too high. With sufficient search frictions, a low-cost agent will match with a high-cost agent (non-assortative matching). Assume further, for simplicity, that investments are perfectly complementary: so, the ex ante investment by the low-cost agent is completely unproductive in a non-assortative match. Nevertheless, an investment by the low-cost agent enhances his bargaining power in such a match by creating – or increasing the value of – a binding outside option, because the investment increases the value of a match with another low-cost agent who has also invested. This rent-transfer opportunity is privately profitable but not socially beneficial.

Note that overinvestment requires "intermediate" match frictions. For rent transfer to be relevant, match frictions must be high enough to ensure that non-assortative matches occur in equilibrium, but not so high that the outside option of a low-cost agent does not bind in these matches. For the case of a CES revenue function, we provide a complete characterisation of the set of parameter values for which overinvestment can occur. Interestingly, overinvestment is possible for a range of match friction values consistent with average unemployment durations in OECD countries. Numerical simulations also indicate that this set tends to be larger, the higher the proportion of low-cost agents, and the lower the returns to scale in the revenue function.

The empirical implications of the matching model and signalling theory overlap to some extent. According to both, education burns up real resources in the process of redistributing income, implying the private return is higher than the social return. This is consistent with the well-known empirical finding that higher education yields substantial private social return. This is consistent with the well-known empirical finding that higher education yields substantial private returns, yet in cross-country studies, the effect on GDP is weak. In Section 4, it is argued that our model, in some respects, fits the facts better than does signalling theory.

The remainder of the paper is organised as follows. Section 2 presents the model, Section 3 derives the overinvestment results, Section 4 discusses the empirical implications of the model, Section 5 discusses related literature, and Section 6 concludes.

2. The model

2.1. Preliminaries

There are two types of agents: firms and workers. Both are infinitely lived. Time $t$ is discrete, with a period length of $\Delta$, and runs infinitely forward and back, and all agents have a discount factor $\delta = e^{-rt}$. The following events occur in each period $t$. First, a measure $\lambda_t$ of both workers and firms of type $i = h, l$ enter the pool of unmatched, with $\lambda_h + \lambda_l = 1$. Then, on entering, each agent chooses how much to invest. A type $i$ agent has investment cost $c_i(e) = c_i e^t$, $i = h, l$ and $\delta_t h > c_t$, all $e$. So, $h$-types have a higher cost of investment than $l$-types.

Then, a fraction $0 < \Delta a < 1$ of the measure of as yet unmatched firms and workers, are randomly matched with each other. That is, every worker is matched with a firm (and vice versa) with probability $\Delta a$. A firm which has invested $e_w$...
and a worker which has invested \( e_f \) can, once matched, produce present value of revenue of \( y(e_w, e_f) \). If both firm and worker are matched, they decide simultaneously and independently whether to accept or reject the match. Should one or both reject, then nothing further happens to these agents until the next period.

If they both accept, both permanently exit the pool of unmatched, and start production. Rather than model bargaining over revenue explicitly, we just assume the outside option principle applies (Osborne and Rubinstein, 1990). That is, revenue \( y(e_w, e_f) \) is equally divided unless half of this quantity is strictly less than the payoff to continued search \( (\text{say } v) \) for one of the two parties, in which case that party gets \( v \) and the other the residual \( y(e_w, e_f) - v \). Such a division can be shown to be an equilibrium of an explicit alternating offers bargaining game between the worker and the firm, where the responder has the option of returning to the pool of the unmatched.\(^9\) Payoffs are assumed linear in revenues, so without loss of generality, these quantities are also the payoffs of the two bargainers.

2.2. The revenue function

We assume that \( y(e_w, e_f) \) is non-negative, symmetric, and strictly increasing, strictly concave and twice differentiable for all \((e_w, e_f) \in \mathbb{R}^2_+\), with \( y_i \) denoting the derivative with respect to the \( i \)th argument. Moreover, we will assume that \( y \) is supermodular: given differentiability, this is just the condition \( y_{12} > 0 \). That is, the inputs are strict complements. One class of functions that satisfies all of these assumptions is the symmetric CES revenue function

\[
y(e_w, e_f) = (0.5e_w^\rho + 0.5e_f^\rho)^{\alpha/\rho} + y_0, \quad \rho < 1, \quad 1 > \alpha > \rho
\]

Note that if \( \rho \leq 0 \), inputs are essential i.e. \( y(0, e) = y(e, 0) = y_0, \) all \( e \). Moreover, \( \rho = 0 \) is the Cobb–Douglas case \( y(e_w, e_f) = e_w^{\alpha/2} e_f^{\alpha/2} + y_0 \).

2.3. Strategies and equilibrium

Section 2.1 above describes a simple stochastic game. We focus on symmetric steady-state Markov-perfect equilibrium of this game. By symmetry, we mean an agent of a given cost type behaves in the same way, irrespective of whether he is firm or worker. The Markov property is simply that agents condition their actions only on payoff-relevant state variables, further discussed below. The steady-state assumption says that in equilibrium, inflows to the pool of unmatched equal outflows for each cost type. Under these assumptions, if a firm (or worker) of cost type \( i = l, h \) enters the market at \( t \), he invests some \( e_i^* \) in equilibrium, independently of \( t \). So, if a firm \( f \) and a worker \( w \) are matched at the beginning of period \( t \), the only payoff-relevant variables for this pair are (i) their two investment levels \( e_w, e_f \); (ii) the distribution of equilibrium investments across all as yet unmatched agents, which is characterised by a pair \((e^*_w, e^*_f)\). Perfection, or sequential rationality, implies that an agent accepts a match at any date if doing so gives a higher payoff than continued search.

3. Overinvestment results

To generate conditions under which overinvestment occurs, we construct a (symmetric steady-state Markov-perfect) equilibrium where (i) matching is non-assortative (NAM) i.e. where an \( l \)-type firm accepts a match with an \( h \)-type worker and vice versa and (ii) where, when an \( h \)-type matches with an \( l \)-type, the outside option of the \( l \)-type binds. Call such an equilibrium an \( N-B \) equilibrium. Both non-assortative matching and a binding outside option are required in equilibrium for overinvestment to occur, for reasons discussed in the introduction.

3.1. The \( N-B \) equilibrium

In an \( N-B \) equilibrium, the present values of expected payoffs to continued search for the two types, denoted \( v_h, v_l \), satisfy the following dynamic programming equations in the limit as \( \Delta \rightarrow 0 \):

\[
rv_l = \lambda_l (v_l - v_l) + a_l (\frac{y(e_l, e_l)}{2} - v_l)
\]

\[
rv_h = \lambda_h (\frac{y(e_h, e_h)}{2} - v_h) + a_l (y(e_l, e_l) - v_l - v_h)
\]

where \( \lambda_l, \lambda_h \) are the shares of low-cost and high-cost agents in the pool of unmatched. Note that due to non-assortative matching, these are the same as the shares of the two types that enter the pool of the unmatched in any period.

Each of these equations has the usual interpretation that the return to being unmatched \((rv_l, rv_h)\) is equal to the expected capital gain from being matched over a short time interval. In the case of the low-cost type, the capital gain is zero when matched with an \( h \)-type (which occurs with probability \( \Delta a\lambda_h \)) because the \( l \)-type’s outside option binds, so he

\(^9\) See an earlier version of this paper, de Meza and Lockwood (2004), for a demonstration of this.
gets payoff \( v_h \), just the value of being unmatched, but if he is matched with another \( l \)-type, the capital gain is \( \frac{y(e_l, h)}{2} - v_l \), as the revenue is equally divided in this case. Eq. (3.2) for the \( h \)-type has a similar interpretation, noting that when matched with an \( l \)-type (which occurs with probability \( \Delta a_{l} \)) the \( h \)-type is residual claimant and thus gets \( y(e_h, l) - v_l \).

Solving (3.1), (3.2) for \( v_l, v_h \) we get

\[
v_l = \phi_l \frac{y(e_l, e_l)}{2}
\]

\[
v_h = \phi_l \left[ \frac{y(e_h, e_h)}{2} + \lambda_l \left( y(e_h, e_l) - \phi_l \frac{y(e_l, e_l)}{2} \right) \right]
\]

where \( \phi_l = \frac{a_l}{\tau + a_l} \). Finally, a binding outside option for the \( l \)-type in an \( hl \) match and NAM respectively require:

\[
v_l > \frac{y(e_h, e_l)}{2}
\]

\[
y(e_h, e_l) \geq v_h + v_l
\]

So, given investments, \( e_h, e_l \), (3.3)–(3.6) fully characterise the N–B equilibrium.

It remains to find the equilibrium investments. Suppose that an individual \( l \)-type agent deviates by a small amount from equilibrium investment \( e_l^* \) to \( e' \). Then, as his outside option continues to bind in a match with an \( h \)-type (for a small enough deviation), his payoff net of investment costs is

\[
\phi_l \frac{y(e', e_l^*)}{2} - c_l e'
\]

So, the equilibrium investment must maximise this expression i.e. must satisfy the first-order condition

\[
\frac{\phi_l}{2} y_1 (e_l^*, e_l^*) = c_l
\]

where \( y_1 \) denotes the first derivative of \( y \). By the same argument, if an individual \( h \)-type agent deviates by a small amount from equilibrium investment \( e_h^* \) to \( e' \), he is still residual claimant in a match with an \( h \)-type (for a small enough deviation), so his payoff net of investment costs is

\[
\phi_h \left[ \lambda_h \frac{y(e', e_h^*)}{2} + \lambda_l \left( y(e', e_l^*) - \phi_l \frac{y(e_l, e_l)}{2} \right) \right] - c_h e'
\]

So, the equilibrium investment must maximise this expression i.e.

\[
\frac{\phi_h \lambda_h}{2} y_1 (e_h^*, e_h^*) + \phi_h \lambda_l y_1 (e_h^*, e_l^*) = c_h
\]

Eqs. (3.8), (3.10) are thus the first-order necessary conditions for equilibrium investments.

However, some discussion of sufficient conditions is required. By assumption, \( y \) is strictly concave in investments, so this might appear to ensure that (3.8), (3.10) are also sufficient. But, there is the additional complication that large deviations in \( e \) away from the equilibrium level can cause the “regime” facing the deviant to change e.g. whether or not he faces a binding outside option in a given kind of match. For example, if the \( l \)-type chooses an \( e' \) sufficiently below \( e_l^* \), he will face first a binding outside option in a match with another \( l \), then as \( e' \) falls further, he will face a binding outside option in a match with an \( h \)-type, \( l \)-types will reject a match with the deviant, etc.

However, it is possible to show the following.\(^{10}\) For the \( l \)-type, all these regime changes make the deviant worse off, so his payoff to (downward) deviation must be bounded above by (3.7). So, if \( e_l^* \) maximises (3.7), it must certainly be a global maximum for the \( l \)-type. A similar argument implies that an \( h \)-type’s payoff to (upward) deviation must be bounded above by (3.9). So, if \( e_h^* \) maximises (3.9), it must certainly be a global maximum for the \( h \)-type. So, to conclude, the N–B equilibrium is fully characterised by \((v_l, v_h, e_l^*, e_h^*)\) that satisfy (3.3), (3.4), (3.5), (3.6), (3.8), (3.10).

### 3.2. Overinvestment

As payoffs are linear in revenue shares, the natural efficiency criterion is the sum of the payoffs to search net of investment costs, which we call aggregate surplus. In N–B equilibrium, aggregate surplus can be written as:

\[
W (e_h^*, e_l^*) = \lambda_h v_h + \lambda_l v_l - \lambda_l c_l e_l^* - \lambda_h c_h e_h^* \\
= \lambda_h \phi_l \left[ \lambda_h \frac{y(e_h^*, e_h^*)}{2} + \lambda_l \left( y(e_h^*, e_l^*) - \phi_l \frac{y(e_l^*, e_l^*)}{2} \right) \right] + \lambda_l \phi_l (1 - \lambda_h \phi_l) \frac{y(e_l^*, e_l^*)}{2} - \lambda_l c_l e_l^* - \lambda_h c_h e_h^*
\]

\(^{10}\) These claims are more formally proved in an online Appendix in the Supplementary material and also available at http://www2.warwick.ac.uk/fac/soc/economics/staff/academic/lockwood.
where in the second line, we have used (3.3), (3.4). Differentiating (3.11) with respect to \( e_l^* \) at \( e_h^*, e_l^* \), and collecting terms, we get:

\[
\frac{1}{\lambda_l} \frac{\partial W(e_h^*, e_l^*)}{\partial e_l} = \lambda_h \phi y_2(e_h^*, e_l^*) + (1 - \lambda_h \phi) \phi y_1(e_l^*, e_l^*) - c_l \\
= \lambda_h \phi y_2(e_h^*, e_l^*) + \phi[0.5 - \lambda_h \phi] y_1(e_l^*, e_l^*)
\]

(3.12)

where in the second line, we have used (3.8). So, investment of the \( l \)-types is locally too high if the term on the right-hand side of the second line of (3.12) is negative. Inspection of this term, using \( y_1(e_l^*, e_h^*) = y_2(e_h^*, e_l^*) \) from symmetry of the revenue function, gives the following result.

**Proposition 1.** In N–B equilibrium, \( e_l^* \) is locally too high i.e. \( \frac{\partial W(e_l^*, e_l^*)}{\partial e_l} < 0 \), iff (i) \( \lambda_h \phi > 0.5 \); (ii) investments are sufficiently complementary i.e.

\[
\frac{y_1(e_l^*, e_l^*)}{y_1(e_l^*, e_h^*)} > \frac{\lambda_h \phi}{\phi[0.5 - \lambda_h \phi]}
\]

(3.13)

The reason why condition \( \lambda_h \phi > 0.5 \) is required is fairly intuitive. First, it is more likely to hold, the higher \( \lambda_h \), as then, the higher is the negative fiscal externality imposed on the \( h \)-types by \( l \)-types’ choice of \( e_l \). Second, it is more likely to hold the higher \( \phi \), as the higher \( \phi \), the more likely are \( h l \) matches.

Condition (3.13) can be explained and interpreted as follows. First, it is easy to show that \( e_l^* > e_h^* \) in equilibrium.\(^{11} \) So, as \( y_1 > 0 \) by assumption, \( y_1(e_l^*, e_l^*) > y_1(e_h^*, e_h^*) \). Now, as \( \phi < 1 \), the right-hand side of the inequality in (3.13) is, by definition, strictly greater than 1. So (3.13) simply requires that \( y_1(e_l^*, e_l^*) \) be sufficiently larger than \( y_1(e_l^*, e_l^*) \) i.e. complementarity in investments has to be sufficiently strong for overinvestment. The intuition for this is as follows. For the standard hold-up reason, we know that there is too little investment in an \( l \) match, so overinvestment requires that the marginal product of \( e_l \) must be low enough in an \( h l \) match. As the only difference between the two types of match is that \( e_l \) is lower, the complementarity requirement follows.

So, generally, an overinvestment equilibrium will exist if (i) all the conditions for N–B equilibrium exist, and (ii) the conditions in Proposition 1 hold. We now wish to obtain conditions in terms of underlying model parameters for which these conditions hold simultaneously. To do this, some simplifying assumptions are required. We make two such assumptions.

A1. \( h \)-types face a cost of investment that is prohibitively costly i.e. \( c_h = \infty \).

A2. The revenue function is CES with \( \rho \leq 0 \).

The main simplification with A1, A2 is the following. First, A1 implies \( e_h^* = 0 \), and A2 then implies that \( y_1(e_l^*, e_l^*) = y_1(e_l^*, 0) = 0 \), as \( y(e, 0) = y_0 \), all \( e \). Thus, (3.13) in Proposition 1 automatically holds, and so overinvestment occurs, as long as \( \lambda_h \phi > 0.5 \). Next, as \( y_1(e_l^*, e_l^*) = y_0 \), conditions (3.5)–(3.6) reduce to the condition that \( y_0 \) in (2.1) lie in a certain interval. Finally, given the assumption of CES revenue, \( e_l^* \) can be solved for explicitly. These facts can all be combined (see Appendix) to prove the following result:

**Proposition 2.** Assume A1, A2. Then, if \( \lambda_h \phi > 0.5 \), and

\[
b^+ = \frac{\kappa \phi_1^{1/(1 - \alpha)}}{(1 - \phi_1)} > y_0 c_1^{\alpha/(1 - \alpha)} > \frac{\kappa \phi_1^{1/(1 - \alpha)}}{1 - 0.5 \theta \phi_1} = b^-
\]

(3.14)

where \( \theta = \frac{1 - \phi + \phi \lambda}{1 - \phi + 0.5 \phi \lambda} \), \( \kappa = \left( \frac{2}{\alpha} \right)^{\alpha/(1 - \alpha)} > 0 \), then there exists an overinvestment equilibrium. There is always a non-empty set of parameters for which there exists an overinvestment equilibrium. For any fixed values of the other parameters, an overinvestment equilibrium exists if match frictions are “intermediate” i.e. \( \phi \) lies in an interval strictly in \([0, 1]\).

Note that the conditions for existence of an overinvestment equilibrium are given entirely in terms of parameters \( \phi \), \( \lambda_h \) (or \( \lambda_l \)), \( \alpha \), \( c_l \); in particular, the precise elasticity of substitution between inputs, \( \rho \), does not matter, as long as it is non-positive. The concavity of the production function, as measured by \( \alpha \), does matter, however.

Proposition 2 says that overinvestment equilibrium always exists for some set of parameters. To get us a feel for how big this set of parameters is, we turn to numerical simulations. Our numerical simulations, reported in Fig. 1 below, proceed as follows.

First, we wish to choose ranges of parameter values which are “realistic” for our main application, the labour market. We begin with the match friction parameter, \( \phi = a/(a + r) \). In the model, in steady-state equilibrium, \( a \) is by definition equal

\(^{11} \) For suppose not i.e. \( e_l^* \leq e_h^* \), but continue to assume that \( l \)-types have a binding outside option i.e. (3.5) holds. Then \( y(e_l^*, e_l^*) < y(e_h^*, e_l^*) \), and moreover, \( v_l \) must be less than (due to match frictions) a weighted average of \( 0.5 y(e_l^*, e_l^*) \) and \( v_l \) itself. Thus, \( v_l < 0.5 y(e_l^*, e_l^*) \), contradicting (3.5).
Fig. 1. Parameter values for which overinvestment equilibrium exists.

to the flow (over time period $\Delta$) of agents exiting from the pool of unmatched, divided by the stock of unmatched. So, $1/a$ is equal to the average time to find a match. Empirically, this corresponds to average unemployment duration. Typical unemployment durations measured in months for OECD countries range between 14 months (France) and 3 months (the US), with an average for Europe of about 6 months (Pissarides, 2007). Taking the time period $\Delta$ as a month, this suggests a range of values for $a$ of $1/14$ to $1/3$.

Next, following Pissarides (2007), we choose an annual discount rate of 5%, implying a monthly discount rate of $\frac{\ln(1.05)}{12} = 0.0041$. Overall, this gives a range of values of $\phi$, 0.934–0.987, i.e. indicating that according to this measure, the labour market is close to frictionless. So, we shall let $\phi$ range between 0.9 and 1. Nevertheless, as we shall see, for reasonable values of the other parameters, it is possible to find overinvestment.

The other parameters here are $\alpha$, the returns to scale of the production function, and $\lambda_h$. These are much more difficult to calibrate from labour market data. First, on $\alpha$, even if there are constant returns to all inputs, the investments considered
here may be only small subset of inputs (e.g. investment in IT training by workers, and capital investments complementary to IT training, for example, computer hardware, by firms) so α could be quite small. We let α take on the value 0.5. Finally, for λ_h, we need λ_h > 0.5/φ for φ ≥ 0.9, i.e. λ_h > 0.55. So, we let λ_l = 1 − λ_h range between 0.04 and 0.44.

Figs. 1(a)–(c) graph b^−, b^+ as functions of the match friction parameter φ = Δf_1/Δf_2, which must lie between 0.9 and 1. Specifically, Figs. 1(a)–(c) show φ along the horizontal axis, and b^−, b^+ on the vertical axis. The set of parameter values satisfying (3.14) is shown by the shaded area in each case. So, from Proposition 2, the set of parameter values for which overinvestment equilibrium exists is just the shaded area. Generally, we see that for every configuration of parameter values illustrated, this set is non-empty. In several cases, this region is quite large, verifying our claim that overinvestment is a realistic possibility in the labour market.

Figs. 1(a)–(c) also show what happens as the fraction of low-cost investors λ_l increases. This clearly increases the size of the shaded area. This is because the higher λ_l, the higher the value of the outside option of an l-type when matched with an h-type, and so the more likely it is that the outside option of an l-type will bind in a match with an h-type i.e. that equilibrium condition (3.5) holds. Other things equal, this makes an overinvestment equilibrium more likely.

Finally, we turn to the important fact, stated in Proposition 2, that for any fixed values of the other parameters, an overinvestment equilibrium exists if match frictions are intermediate i.e. φ is in an interval strictly in [0, 1]. First, how can that be reconciled with Fig. 1, where φ can take on very high values? Simply fix a point on the vertical axis and there is always an interval on the horizontal axis that generates coordinates in the shaded area. So, "intermediate" in Proposition 2 has a very precise meaning; it does not mean, for example, that values of φ in the middle of the feasible range [0, 1] i.e. around 0.5 always generate overinvestment.

Second, why are "intermediate" frictions required for overinvestment? The reason is that an N–B equilibrium only occurs with intermediate match frictions. Match frictions must be low enough to ensure that the l-type's outside option is binding, but must be high enough to ensure that matching is non-assortative. Moreover, the only way in which overinvestment can arise in N–B equilibrium, and thus a necessary condition for overinvestment is that match frictions are intermediate.12

4. Discussion

Here we discuss some possible extensions and empirical implications of the model. One concern is that in the model, the matching process is entirely random, whereas in reality, labour market search is at least partially "directed" i.e. workers can apply to particular firms, and firms can accept applications only from particular workers. Here, we sketch how our model can be extended to allow for directed search,13 and argue that our results are robust to a certain amount of “direction” in the search process.

An interpretation of our matching technology is that over a time interval Δ, a fraction of Δa of firms and workers are drawn at random from the pool of unmatched, and then matched randomly with each other via an employment agency of some kind. We now modify this as follows. We suppose that low-cost types can express a preference to the agency for the type they wish to be matched with. An l-type will always wish to be matched with another l-type, as strict complementarity implies that total revenue from the match will be higher, and thus half the revenue from that match exceeds what the l-type could get in a match with an h-type, whether or not his outside option binds. We also suppose that the agency only meets the l-type's request with probability p, and matches him randomly with probability 1 − p, with p thus measuring the efficiency of the agency.

So, the probability, conditional on being matched at all, that an l-type is matched with another l-type is θ_l = p + (1 − p)λ_l, and the probability, conditional on being matched at all, that an h-type is matched with another h-type is θ_h = λ_h + pλ_l. So, when p = 0, we have random matching, and when p = 1, we have θ_l = θ_h = 1 i.e. perfectly directed matching. Then, in (3.1), we replace λ_l, λ_h by θ_l, 1 − θ_l respectively and in (3.2), we replace λ_l, λ_h by θ_h, 1 − θ_h respectively. Then, the analysis proceeds as before, and it can be shown (details on request) that if p is not too high, an overinvestment equilibrium can exist as before. In this sense, our results are robust to the introduction of directed search.

Now we turn to empirical implications. In some respects, our matching model fits the facts better than does signalling theory. For example, Sicherman (1991, p. 114) reports that "Workers who are working in occupations that demand less schooling than they actually have (overeducated) get higher wages than their co-workers (holding other characteristics constant) but lower wages than workers with similar levels of schooling who work in jobs in which their schooling equals that which is required." This is exactly the pattern predicted by the matching approach. Moreover, signalling theory is based on the proposition that education sorts, whereas the dispersion in matches indicates that to a considerable extent it fails to do so.

The theories also have different implications for how the growth in the numbers of graduates impacts on the level of wages. Acemoglu and Shimer (1999) document that in the US and elsewhere that an upsurge in the number of graduates

12 A formal proof that underinvestment occurs with any other kind of equilibrium is provided in de Meza and Lockwood (2004). The idea is that there are only two other possibilities: (i) assortative matching, where by definition, the outside option cannot bind, and (ii) non-assortative matching, with non-binding outside options. In either case, there is no rent-shifting incentive for investment. In addition, if the return to the unmatched state occurs randomly via stochastic match break-up in equilibrium, there is always underinvestment.
13 For other models of directed search, see e.g. Moen (1997), Albrecht et al. (2006).
has been associated with falls in the absolute wage of non-graduates.\footnote{Acemoglu’s explanation is that with more graduates available, firms find it worth creating jobs specifically for graduates. The implication is that there should be a strong positive correlation between the number of graduates and GNP.} If we interpret the ex ante investment in our model as the acquisition of higher education, our rent-transfer effect provides an explanation of the facts that does not rely on asymmetric information. In our model, an increase in the number of low-cost investors (those willing and able to invest in higher education) increases the outside option of these investors and thus reduce the wages of non-investors (non-graduates). So the absolute wage of non-graduates will be lower in an equilibrium with high numbers of graduates, as the evidence suggests. In contrast, the separating equilibrium of signalling theory implies that the least educated are paid their intrinsic productivity, so an increase in the number of more educated workers would not depress their wages.

5. Related literature

We are aware of two papers that consider ex ante investments followed by competitive, or frictionless, mechanisms for pairing or matching agents (e.g. firms and workers): Cole et al. (2001) and Felli and Roberts (2002).\footnote{Cole et al. (2001) is less closely related to our work, as it assumes a finite number of agents. In this case, the decision of any two agents to match has an external effect on the opportunities available to other agents, so the set-up is a bit different.} Cole et al. (2001) consider a matching model in which buyers and sellers make investment decisions non-cooperatively prior to entering a frictionless matching and bargaining process that is modelled as a cooperative game. The outcome of this second stage is constrained to be “stable” i.e. there is no pair of agents that by rematching and appropriately sharing the resulting surplus can both be strictly better off than in the equilibrium. Felli and Roberts (2002) analyse a frictionless model with a fixed number of heterogeneous buyers and sellers, and investment only by one side of the market. Following investment, a Bertrand-style game is assumed where firms bid for workers (or vice versa). Both these papers find examples of equilibria with overinvestment.

But, we would argue, these examples have important limitations. In Felli and Roberts (2002), unlike in our model, the agents invest efficiently, conditional on the match that they anticipate. But, relative to the first-best, overinvestment by – for example – a relatively low quality worker is possible, because he anticipates being hired by a very high-quality firm. There is no general tendency to overinvestment or underinvestment.

In Cole et al. (2001) Proposition 5 states that equilibrium investments with stable matching are at a local maximum of net surplus (the revenue from a match minus investment costs, \(S = y(e_w, e_f) - c(e_w) - c(e_f)\) in our notation). So, if \(S\) is concave, the unique equilibrium investments with stable matching are the efficient investments that maximise \(S\). So, their example of overinvestment with a continuum of agents relies on \(S\) being non-concave.\footnote{In our notation, their example is \(y(e_w, e_f) = \begin{cases} e_w e_f, & e_w e_f \leq 0.5 \\ 2(e_w e_f)^2, & e_w e_f < 0.5 \end{cases}\) which is clearly not concave.} In contrast, our model has a well-behaved concave revenue function; overinvestment is due to a different mechanism.

Moreover, in their model, typically there are multiple stable equilibria to the postinvestment game, and thus multiple equilibria overall, which may generally involve underinvestment, efficient investment, or overinvestment. In their cooperative framework, without an explicit bargaining model, they have no criterion for selecting among these equilibria.

6. Conclusions

A recent literature examines agents’ incentives to make investments prior to entering a stochastic matching process and bargaining over the surplus. In these models outside options do not bind, investors are held up and thus will underinvest in equilibrium. We have shown that this finding is not robust. If agents are heterogeneous, outside options influence investment incentives. This effect mitigates underinvestment, and even allows overinvestment to arise. Simulation results indicate that empirically, parameters may be in the overinvestment zone. Unlike the signalling model, the matching model is consistent with findings that workers often take jobs for which they are overqualified, earning more than less qualified peers but less than if they were to hold jobs for which their qualifications are required.

Appendix. Proof of Proposition 2

Note that as \(y(e_w, e_f) = y(0, e_f) = y_0\), conditions (3.5) reduce to \(2v_l > y_0\). Moreover, using (3.3), (3.4), (3.6) can be written

\[
y_0 \geq \left( \frac{1 - \phi + \phi \lambda_h}{1 - \phi + 0.5 \phi \lambda_h} \right) v_l \equiv \theta v_l \tag{A.1}
\]

So, from (A.1), the condition for the binding outside option and non-assortative matching conditions to be satisfied together become

\[
2v_l > y_0 \geq \theta v_l \tag{A.2}
\]
Now from the CES assumption (2.1), we have
\[ y(e_l, e_l) = e_l^\alpha + y_0 = A + y_0 \]  \hspace{1cm} (A.3)
Combining this with (3.3) gives \( v_l = 0.5\phi_l(y_0 + A) \). But then, assuming \( y_0 > 0 \), (A.2) reduces to
\[ \frac{\phi_l A}{1 - \phi_l} > y_0 > \frac{0.5\phi_l A}{1 - 0.5\phi_l} \]  \hspace{1cm} (A.4)
Finally, \( e_l \) satisfies the FOC (3.8):
\[ \frac{\alpha\phi_l}{2}(e_l^\alpha)^{\alpha/\rho - 1}e_l^{\alpha/\rho - 1} = c_l \implies e_l^* = \left( \frac{\alpha\phi_l}{2c_l} \right)^{1/(1-\alpha)} \]
So,
\[ A = (e_l^*)^\alpha = \kappa \left( \frac{\phi_l}{c_l} \right)^{\alpha/(1-\alpha)} \quad \text{and} \quad \kappa = \left( \frac{\alpha}{2} \right)^{\alpha/(1-\alpha)} > 0 \]  \hspace{1cm} (A.5)
and thus, combining (A.4), (A.5), we need
\[ \frac{\kappa \phi_l^{1/(1-\alpha)}}{(1 - \phi_l)c_l^{\alpha/(1-\alpha)}} > y_0 > \frac{0.5\phi_l^{1/(1-\alpha)}}{(1 - 0.5\phi_l)c_l^{\alpha/(1-\alpha)}} \]
which gives (3.14). Finally, we need to prove that for any fixed values of the other parameters, an overinvestment equilibrium exists if match frictions are intermediate i.e. \( \phi \) is in an interval strictly in \([0, 1]\). First, as \( \phi \to 1 \), \( 0.5\theta \), \( \phi_l \to 1 \) also, so that \( b^+, b^- \to \infty \), so that for \( \phi \) close enough to 1, any fixed \( y_0c_l^{\alpha/(1-\alpha)} < b^+ \), so no overinvestment equilibrium can exist for \( \phi \) close enough to 1. Second, as \( \phi \to 0 \), \( \phi_l \to 0 \), so \( b^+, b^- \to 0 \), so that for \( \phi \) close enough to 1, any fixed \( y_0c_l^{\alpha/(1-\alpha)} > b^+ \), so no overinvestment equilibrium can exist for \( \phi \) close enough to 0. \( \Box \)

**Supplementary material**

The online version of this article contains additional supplementary material. Please visit DOI: 10.1016/j.jgeb.2009.11.005.

**References**

De Meza, D., Lockwood, B., 2006. Ex ante investments and outside options: some recent results reconsidered. Available at: http://www2.warwick.ac.uk/fac/soc/economics/staff/faculty/lockwood/.