Revenue Deficiency under Second-Price Auctions in a Supply-Chain Setting

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Abstract

Consider a firm, called the buyer, that satisfies its demand over two periods by assigning both demands to a supplier via a second-price procurement auction; call this the *Standard auction*. In the hope of lowering its purchase cost, the firm is considering an alternative procedure in which it will also allow bids on each period individually, where there can be either one or two winners covering the two demands; call this the *Multiple Winner auction*. Choosing the Multiple Winner auction over the Standard auction can in fact result in a higher cost to the buyer. We provide a bound on how much greater the buyer's cost can be in the Multiple Winner auction and show that this bound is tight. We then sharpen this bound for two scenarios that can arise when the buyer announces his demands close to the beginning of the demand horizon. Under a monotonicity condition, we achieve a further sharpening of the bound in one of the scenarios. Finally, this monotonicity condition allows us to generalize this bound to the T-period case in which bids are allowed on any subset of period demands.

Keywords: procurement; supply chain; second-price auction; VCG mechanism; revenue deficiency

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1 Introduction

Consider a firm, called the *buyer*, having demand over two periods for a product upstream in the supply chain. The buyer assigns both period demands to a supplier via a second-price procurement (reverse) auction, i.e., a Vickrey auction (Vickrey, 1961). The winner will be the supplier who submits the lowest bid, and he will be paid by the buyer not his bid but the second-lowest bid. We will refer to this as the *Standard auction*. Under normal conditions, the suppliers will be given sufficient advance notice by the buyer such that, in formulating their bids, the suppliers can each assume that they will have full use of their respective production capacity.

In the hope of lowering its purchase cost, the buyer is considering allowing bids on each period demand individually in addition to the package of the two demands. Period demand is considered to be an indivisible item. In this auction, called the *Multiple Winner auction*, clearly there can be two winners covering the demands rather than just one. Here, the buyer assigns orders to his suppliers via the generalization of the Vickrey auction to heterogeneous goods, the *VCG mechanism* (Ausubel and Milgrom, 2006). Under the VCG mechanism, each supplier reports to the buyer his costs of supplying each possible subset of the buyer's desired collection of items. The buyer then combines all information from all of the bidders to determine the optimal allocation, i.e., the one in which the buyer's costs are minimized, which will impute who the winning bidders will be, where each winning bidder will be paid not his bid but the incremental surplus that he brings to the auction.

However, as is well-known, the VCG mechanism suffers from a weakness called *revenue deficiency* (Ausubel and Milgrom, 2006; Rothkopf, 2007; Conitzer and Sandholm, 2006). Ausubel and Milgrom (2006) provide a simple hypothetical example, originally presented in Ausubel and Milgrom (2002), involving a forward auction of two items to three bidders. Bidder 1 only desires the package of two items and it is willing to pay $\alpha > 0$, whereas bidders 2 and 3 are both willing to pay the same price for any of the two single items. The VCG mechanism assigns the items to bidders 2 and 3 who both pay zero. Ausubel and Milgrom point out that if the two items were instead auctioned as an indivisible set, then there would be three bidders each willing to pay α for the set. The winner would be any of them and would be paying α, yielding a higher revenue than in the VCG mechanism.

The revenue deficiency phenomenon can arise in a procurement environment as well, as illustrated by the following example. A buyer needs to satisfy demand over two periods, where the demands in the two periods are identical. There are four possible suppliers, where none has significant holding costs. Suppliers 1 and 2 both have sufficient capacity in period 1 to satisfy the sum of the demands in periods 1 and 2, and both have sufficient capacity in period 2 to satisfy demand in period 2; both suppliers face a setup cost of $f > 0$ in each period. Suppliers 3 and 4, due to previous commitments, each have already sunk their setup costs in both periods, but only have remaining capacity in period 1, where this capacity is sufficient to satisfy one period of demand but not both. Thus, only suppliers 1 and 2 can participate in the Standard auction; one will be chosen the winner and be paid f. All four suppliers can participate in the Multiple Winner auction, suppliers 3 and 4 will be chosen as winners, and both will be paid f. Thus, the buyer will be facing an additional payment of f in the Multiple Winner over the Standard auction.

This example shows that there are cases in which the buyer will be worse off by choosing the Multiple Winner auction, i.e., the buyer needs to pay more to satisfy his demand. In this example, inventory holding costs did not come into play, nor did unit production costs. The goal of this paper is to begin a rigorous investigation of the revenue deficiency phenomenon in a procurement environment that takes account of all the relevant costs. There is scarce analytic work on revenue deficiency. In the context of a procurement auction, revenue deficiency is *cost excess*, and in the sequel we will primarily use the latter term.

Our contribution in this paper is to provide a bound on how much worse off the buyer can be with the Multiple Winner auction, i.e., how large the cost excess can be. (Of course, the cost excess can be negative.) Specifically, we first provide an upper bound on the amount by which the cost to the buyer in the Multiple Winner auction can exceed his cost in the Standard auction. Next, we show that this bound is tight. Then we sharpen this bound for two scenarios that can arise when the buyer announces his demands close to the beginning of his demand horizon, and thus the suppliers have already made some commitments with other buyers. In the first scenario, each supplier is already committed to producing in both periods but may have some spare production capacity in one or both periods; in the second scenario, each supplier has available capacity in only one period. We achieve a further sharpening of the bound in the second scenario when a mild monotonicity condition is satisfied. Finally, this monotonicity condition allows us to generalize the bound to the T-period case in which bids are allowed on any subset of periods.

The remainder of our paper is organized as follows. In the next section, we review the literature on procurement auctions. In Section 3, we formally introduce the two procurement auctions that we will be considering in the paper. In Section 4, we compare the buyer's purchase cost resulting from running a Multiple Winner auction versus running a Standard auction, and then consider the two scenarios described above. In Section 5 we introduce the monotonicity condition and the additional results that can be achieved when the condition holds. In Section 6 we present our conclusions. All proofs appear in the Appendices.

2 Literature review

There exists a considerable literature on procurement auctions. In this section, we review the work on procurement auctions most relevant to the setting considered in this paper, which focuses on supply chain costs, see e.g. Teich et al. (2006) and references therein for multi-attribute auctions and e.g. Lorentziadis (2012) and references therein for auctions of mixed populations of bidders. An exposition of procurement auctions is provided by Bichler et al. (2006), including a description of an industrial procurement auction conducted at Mars, Inc., which was presented by Hohner et al. (2003). Tunca and Wu (2009) and Olivares et al. (2012) provide a number of examples of companies and government organizations that make use of procurement auctions, including SUN Microsystems, Hewlett-Packard, IBM, Samsung, and Lucent.

Wan and Beil (2009) consider a manufacturer using a reverse auction in combination with supplier qualification screening to determine which qualified supplier will be awarded a contract. They analytically explore the trade-offs between varying the levels of prequalification and postqualification. Kostamis et al. (2009) consider a choice between two auction formats where the suppliers know only their own true production cost, while the buyer does not know the suppliers' true production costs but does have some limited information in that regard. Chen and Vulcano (2010) consider first- and second-price auctions in the context of a single supplier who auctions his capacity to two re-sellers, one of whom will have his bid revealed, and one who will keep his bid hidden after the auction. Comparing the supply chain performance under both auction formats, the authors find that the second-price auction leads to higher payoffs for all parties.

These aforementioned papers do not address the multi-period auction case. There are not many papers on this topic. In fact, in the procurement auction literature, the phrase "multi-period procurement" almost invariably refers to auctioning items *sequentially* via a series of single-period auctions, as in Elmaghraby (2003), rather than auctioning them *simultaneously* via a single auction for multiple periods, as we do here.

One exception is Kameshwaran et al. (2005), who touch on this idea in passing. They consider the procurement of heterogeneous items for a single period, where each bidder submits a single *discount bid* consisting of the cost for each item it offers to supply together with a discount based on the number of items actually supplied. The authors point out, however, that in the multi-period case these discount bids would not be appropriate, and that if period demand is considered to be an indivisible item, then the problem reduces to procurement of multiple items where a package bid can express a supplier's cost function more efficiently.

A line of research related to the setting considered in this paper is presented in Elmaghraby (2005). In her model, a buyer seeks to purchase two units, and auctions off the second unit after the winner of the first auction has been announced. She is interested in how suppliers bid in the presence of competitors with asymmetric production capacity. She assumes that there are two types of bidders: "global bidders," who have sufficient capacity to supply both units, and "small bidders," who can supply only one unit. Elmaghraby performs extensive numerical analysis and concludes that adding small bidders to the sequential auction with only global bidders may either increase or decrease the expected procurement costs. Her innovative work opens up an interesting line of enquiry regarding the relationship between the production capacities of suppliers and the procurement cost of the buyer. As pointed already, Elmaghraby's setting is different than ours, since she considers a sequential auction rather than a multi-period auction.

3 The two auctions

In this section we introduce the two procurement auctions that we will be comparing in the paper. In both the Standard auction and the Multiple Winner auction, the buyer announces to the set of suppliers $\mathbf{S} = \{1, \ldots, S\}$ his demand requirements (D_1, D_2) over the two periods. Period demand is considered to be an indivisible item, i.e., it has to be delivered by a single supplier satisfying thus the so-called single-sourcing property. Supplier production in a period occurs at the beginning of that period. We assume that delivery in a period is received sufficiently early that it is available for use in that period. The suppliers are assumed to have independent private values, i.e., the supplier's payoff depends solely on his own estimate of value and not on the other suppliers' estimates of value.

Supplier Cost Structure and Buyer Cost Structure. Hereafter, we will use the term *supplier's total cost* to refer to the sum of a specified supplier's production and holding costs, while the term *buyer's total cost* will refer to the sum of the buyer's purchase costs and holding costs. More specifically, if supplier i produces in a period t, then it incurs a setup cost f_i and a unit production cost p_i; further, it faces a production capacity of b_{it}. If supplier i carries inventory over from the first period to the second, then it incurs a unit inventory holding cost h_i. The buyer pays each supplier at the end of the auction, which occurs before the beginning of period 1. If the buyer carries inventory, then it incurs a unit inventory holding cost H. Recall the standard supply chain principle that unit inventory holding cost can only increase as one moves down the supply chain (Silver et al., 1998). In our context, this means that the unit holding cost of the buyer is at least as great as at any one of his suppliers, i.e. $h_i \leq H$, for all i. This principle, together with the salient feature of the VCG mechanism that bidders will report their true values, imply that the buyer will prefer that all inventory is kept at the suppliers.

The two auctions are defined as described below.

The Standard Auction. The buyer announces to the suppliers the demands D_1 and D_2. The suppliers are then asked to each submit a bid, viz., a price for supplying both period demands in their respective periods. Let C_i be the bid price submitted by supplier i. Here, the possible bidders are those whose production capacity in period 1 is sufficient to meet buyer demand in period 1, and whose cumulative production capacity in periods 1 and 2 is sufficient to meet cumulative buyer demand in periods 1 and 2. Let w denote the winner of the Standard auction, i.e. the one with the lowest bid price among suppliers in $\mathbf{S}$. Let $w\_$ denote the winner in the Standard auction when the set of suppliers is restricted to $\mathbf{S}\backslash\{w\}$. Let $C_{\mathbf{S}}$ denote the cost to w associated with delivering the demands, i.e., $C_{\mathbf{S}} = \min_{i\in\mathbf{S}} C_i = C_w$.

The *buyer's total cost in the Standard auction*, J, is given by

$$J \;=\; C_{\mathbf{S}\backslash\{w\}} = C_{w\_}. \tag{1}$$

The Multiple Winner auction. The buyer announces to the suppliers the demands D_1 and D_2. The suppliers are then asked to each submit a set of three bids: (i) a price for supplying period 1 demand in period 1, (ii) a price for supplying period 2 demand in period 2, and (iii) a price for supplying both period demands in their respective periods. These bids are considered to be *exclusive-or* (XOR) bids, which means that the auctioneer can accept at most one bid from any given bidder. (See Nisan, 2006, for a discussion of XOR bids and Bichler et al., 2011, for a discussion on bidding languages in the presence of economies of scale and scope.) Note that a bid price can be infinite. Each supplier i is restricted by his production capacity b_{it} in period t. Let w_t denote the winner of demand D_t in the Multiple Winner auction, for $t=1,2$. Let $\mathbf{W} = \{w_1, w_2\}$ be the set of winners in this auction and use W to denote the cardinality of set $\mathbf{W}$, $W = |\mathbf{W}| = 1$ or 2. Note that if $W = 1$, w_1 and w_2 will be identical. Let $C^{\mathrm{M}}_{\mathbf{S}}$ denote the total cost to w_1 associated with delivering demand D_1 and to w_2 associated with delivering demand D_2.

The *buyer's total cost in the Multiple Winner auction*, J^{M}, is given by the buyer's sum total payment to the suppliers:

$$
\begin{aligned}
J^{\mathrm{M}} &= C^{\mathrm{M}}_{\mathbf{S}} + \sum_{i\in\mathbf{W}} (C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - C^{\mathrm{M}}_{\mathbf{S}}) \qquad (2)\\
&= \begin{cases} C^{\mathrm{M}}_{\mathbf{S}\setminus\{w_1\}} + C^{\mathrm{M}}_{\mathbf{S}\setminus\{w_2\}} - C^{\mathrm{M}}_{\mathbf{S}} & \text{when } W=2 \\ C^{\mathrm{M}}_{\mathbf{S}\setminus\{w_1\}} & \text{when } W=1 \end{cases} .
\end{aligned}
$$

The Multiple Winner auction allows for three production possibilities: (i) D_1 and D_2 are produced by a single supplier in period 1, in which case the supplier incurs an inventory holding cost for the second period demand; (ii) D_1 and D_2 are produced by two different suppliers in period 1, in which case the supplier for D_2 incurs a holding cost; or (iii) D_1 is produced in period 1 and D_2 is produced in period 2, which involves one or two suppliers and no holding costs. In the sequel, we will refer to these three as *policy (i)*, *policy (ii)*, and *policy (iii)*, respectively.

We formally define the cost excess as $J^{\mathrm{M}} - J$. We present below an example of cost excess that we will use and modify throughout the paper as a paradigm to illustrate the concepts we develop and prove formally. In the example there are only two suppliers, where supplier 1 does not have any capacity in period 2 and therefore must produce all the demand in period 1. Both suppliers have the same setup costs, where the unit production costs of supplier 1 are the lower of the two. However, when delivering in period 2, supplier 1 must produce in period 1 and pay for holding the demand in inventory, with his total unit cost, p_1+h_1, higher than that incurred by supplier 2, p_2.

Example 3.1 Let the demand in each period be D. There are two suppliers, each of which faces a setup cost of 0 in each period, and a unit holding cost of 0.02. The other supplier data are shown in the table below:

i	f_i	p_i	b_{i1}	b_{i2}
1	0	0.100	$2D$	0
2	0	0.111	$2D$	D

Recall that the setup costs are equal to zero, thus we can focus on the unit costs when discussing all possible production options in both the Standard and the Multiple Winner auctions.

Let us first look at the Standard auction. Due to the lack of capacity in period 2, supplier 1 needs to carry the second demand in inventory and its total cost is equal to

$$C_1 = 0 + 0.10\,(D + D) + 0.02\,(D) = 0.22\,D.$$

On the other hand, supplier 2 has capacity in period 2 and the least costly option consists of producing each demand in its own period

$$C_2 = (0 + 0.111\,D) + (0 + 0.111\,D) = 0.222\,D.$$

Therefore, the winner of the Standard auction is supplier 1 with

$$C_{\mathbf{S}} = 0.22\,D \quad [w\!=\!1].$$

Now let us look at the Multiple Winner auction. Because the setup costs are equal to zero in the Multiple Winner auction, each demand will be allocated to the supplier with the lowest total unit cost. This means that demand D_1 will be allocated to $\arg\min\{p_1, p_2\}$, while demand D_2 will be allocated to $\arg\min\{p_1\!+\!h, p_2\}$, where the total unit cost associated with supplier 1 when delivering demand D_2 is equal to $p_1\!+\!h$ since supplier 1 lacks from capacity in period 2. Since in our problem instance $p_1 < p_2 < p_1\!+\!h$, the Multiple Winner auction will have two winners. Indeed:

$$\begin{aligned} C^{\mathrm{M}}_{\mathbf{S}} &= \min\{0.10, 0.111\}\,D + \min\{0.10 + 0.02, 0.111\}\,D \\ &= 0.10\,D + 0.111\,D = 0.211\,D \quad [w_1\!=\!1,\ w_2\!=\!2]. \end{aligned}$$

Since there are only two suppliers and the winner of the Standard auction is supplier 1, the cost excess is given by $C_1 \!-\! C^{\mathrm{M}}_{\mathbf{S}}$. Indeed,

$$\begin{aligned} J^{\mathrm{M}} \!-\! J &= C^{\mathrm{M}}_{\mathbf{S}\setminus\{1\}} + C^{\mathrm{M}}_{\mathbf{S}\setminus\{2\}} - C^{\mathrm{M}}_{\mathbf{S}} - C_{\mathbf{S}\setminus\{1\}} \\ &= C^{\mathrm{M}}_2 + C^{\mathrm{M}}_1 - C^{\mathrm{M}}_{\mathbf{S}} - C_2 \\ &= C_2 + C_1 - C^{\mathrm{M}}_{\mathbf{S}} - C_2 \\ &= C_1 - C^{\mathrm{M}}_{\mathbf{S}} \end{aligned}$$

Note that in both C_1 and $C^{\mathrm{M}}_{\mathbf{S}}$, supplier 1 delivers D_1, thus the cost excess focuses on D_2

$$= \quad ((0.10 + 0.02)\,D) - (0.111\,D).$$

This means that the cost excess is equal to $0.009\,D$, and therefore positive. $\square$

In the following example, we have three suppliers where one of them only has capacity in period 2. As opposed to Example 3.1, the setup costs are discriminatory.

Example 3.2 Let the demand in each period be $D \in [100, 125]$. There are three suppliers. Supplier 2 and 3 face a setup cost of 2 in each period, while supplier 1 faces a setup cost of 3.9. Each supplier faces a unit holding cost of 0.01. The other supplier data are shown in the table below.

i	f_i	p_i	b_{i1}	b_{i2}
1	3.9	0.09	$2\,D$	0
2	2	0.10	$2\,D$	0
3	2	0.07	0	D

Let us first look at the Standard auction. Due to capacity constraints, only suppliers 1 and 2 can take part in this auction. Due to the lack of capacity in period 2, they both produce all the demand in period 1. The winner of this auction is supplier 1. Indeed:

$$\begin{aligned} C_1 &= 3.9 + 0.09\,(D + D) + 0.01\,(D) = 3.9 + 0.19\,D \\ C_2 &= 2 + 0.10\,(D + D) + 0.01\,(D) = 2 + 0.21\,D. \end{aligned}$$

Since $D \geq 100$, we have that

$$C_{\mathbf{S}} = 3.9 + 0.19\,D \quad [w\!=\!1].$$

Now let us look at the Multiple Winner auction. We may notice that the unit inventory holding costs are the same for all suppliers, and that when capacity is available in period 1, this is large enough to produce both demands in period 1. We conclude that policy (ii) is dominated by policy (i), and only two production possibilities need to be considered in the Multiple Winner auction, policy (i) and policy (iii). Therefore:

$$\begin{aligned} C_{\mathbf{S}}^{\mathrm{M}} &= \min\,\{\min_i\{f_i + p_i(D_1\!+\!D_2) + h_i(D_2)\},\ \min_{i,\,i'}\{(f_i + p_iD_1) + (f_{i'} + p_{i'}D_2)\}\} \\ &= \min\,\{3.9 + 0.19\,D \ \ [w_1\!=\!w_2\!=\!1], \\ &\qquad\qquad \min\{3.9 + 0.09\,D, 2 + 0.10\,D\} + (2 + 0.07\,D)\} \end{aligned}$$

Since $D \leq 125$, we have that

$$\begin{aligned} &= \min\,\{3.9 + 0.19\,D \ \ [w_1\!=\!w_2\!=\!1], \\ &\qquad\qquad (2 + 0.10\,D) + (2 + 0.07\,D) \ \ [w_1\!=\!2,\ w_2\!=\!3]\} \end{aligned}$$

Since $D \geq 100$, we have that

$$= 4 + 0.17\,D \ \ [w_1\!=\!2,\ w_2\!=\!3].$$

Thus, the lowest cost policy is policy (iii), with supplier 2 producing D_1 and supplier 3 producing D_2 in period 2.

The cost excess is given by:

$$\begin{aligned} J^{\mathrm{M}} - J &= C^{\mathrm{M}}_{\mathbf{S}\setminus\{2\}} + C^{\mathrm{M}}_{\mathbf{S}\setminus\{3\}} - C^{\mathrm{M}}_{\mathbf{S}} - C_{\mathbf{S}\setminus\{1\}} \\ &= C^{\mathrm{M}}_{\mathbf{S}\setminus\{2\}} + C^{\mathrm{M}}_{\mathbf{S}\setminus\{3\}} - C^{\mathrm{M}}_{\mathbf{S}} - C_2 \end{aligned}$$

After we eliminate supplier 3, policy (iii) is infeasible and supplier 1 produces both demands

$$\begin{aligned} &= C^{\mathrm{M}}_{\mathbf{S}\setminus\{2\}} + C_1 - C^{\mathrm{M}}_{\mathbf{S}} - C_2 \\ &= (C^{\mathrm{M}}_{\mathbf{S}\setminus\{2\}} - C^{\mathrm{M}}_{\mathbf{S}}) + (C_1 - C_2) \end{aligned}$$

After we eliminate supplier 2, D_1 will be reallocated to supplier 1

$$\begin{aligned} &= ((3.9 + 0.09\,D) - (2 + 0.10\,D)) + (1.9 - 0.02\,D) \\ &= (1.9 - 0.01\,D) + (1.9 - 0.02\,D) \\ &= (3.8 - 0.03\,D). \end{aligned}$$

Since $D \in [100, 125]$, the cost excess $\in [0.05, 0.8]$, and therefore, it is positive. $\square$

As discussed earlier, although revenue deficiency has been observed in other contexts, bounds have rarely if ever been discussed. In the following two sections, we provide upper bounds on the cost excess in the procurement context.

4 Bounding the cost excess

This section is devoted to bounding the cost excess. In Section 4.1, we provide an upper bound on the cost excess of the Multiple-Winner versus Standard auction. This bound is given in terms of the setup costs and the suppliers inventory holding costs. As a straightforward result, we provide an upper bound on the cost excess in terms of the setup costs and the buyer inventory holding costs. In Section 4.2, we show by construction that the cost excess bound is tight. Finally, in Section 4.3, we sharpen this bound for two scenarios involving assumptions on setup costs and production capacities, respectively.

4.1 The cost excess bound

In the following result we present an upper bound on the cost excess for arbitrary demands, costs, and capacities.

Theorem 4.1 *The cost excess is bounded from above as follows:*

$$J^{\mathrm{M}} - J \;\leq\; \max\{\max_{i \in \mathbf{S}} f_i, \max\left\{\frac{D_1}{D_1 + D_2}, \frac{1}{2}\right\}\left[(\max_{i \in \mathbf{S}} h_i) D_2 - \min_{i \in \mathbf{S}} f_i\right]\}. \tag{3}$$

In the previous theorem, the second part of the bound is a fraction of the difference between the largest inventory holding costs associated with demand D_2 and the cheapest setup cost. The following two examples illustrate this part of the bound.

The example below is in line with Example 3.1, though the cost figures are slightly different and $D_1 \leq D_2$.

Example 4.2 Let the demands be $(D-\varepsilon, D)$, where $0 < \varepsilon < D$. There are two suppliers, each of which faces a setup cost of 0 in each period, and a unit holding cost of 0.02. The other supplier data are shown in the table below.

i	f_i	p_i	b_{i1}	b_{i2}
1	0	0.100	$2\,D$	0
2	0	0.110	$2\,D$	D

Let us first look at the Standard auction. As in Example 3.1, supplier 1 needs to carry the second demand in inventory while for supplier 2 the cheapest option consists of producing each demand in its own period, and

$$\begin{aligned} C_1 &= 0 + 0.10\,(D-\varepsilon + D) + 0.02\,(D) \;=\; 0.22\,D - 0.10\,\varepsilon, \\ C_2 &= 0 + 0.11\,(D-\varepsilon) + 0 + 0.11\,(D) \;=\; 0.22\,D - 0.11\,\varepsilon. \end{aligned}$$

Thus, the winner of this auction is supplier 2 with:

$$C_{\mathbf{S}} = 0.22\,D - 0.11\,\varepsilon \quad [w = 2].$$

As in Example 3.1, in the Multiple Winner auction each demand will be allocated to the supplier with the lowest total unit cost. Thus:

$$\begin{aligned} C^{\mathrm{M}}_{\mathbf{S}} &= \min\{0.10, 0.11\}\,(D-\varepsilon) + \min\{0.10 + 0.02, 0.11\}\,(D) \\ &= 0.10\,(D-\varepsilon) + 0.11\,(D) \\ &= 0.21\,D - 0.10\,\varepsilon \;\; [w_1 = 1,\; w_2 = 2]. \end{aligned}$$

Since there are only two suppliers and the winner of the Standard auction is supplier 2, the cost excess is given by $C_2 - C^{\mathrm{M}}_{\mathbf{S}}$, which is equal to the difference in costs incurred when supplying D_1:

$$\begin{aligned} J^{\mathrm{M}} - J &= C_2 - C^{\mathrm{M}}_{\mathbf{S}} \\ &= 0.11\,(D-\varepsilon) - 0.10\,(D-\varepsilon) \\ &= 0.01\,(D-\varepsilon), \end{aligned}$$

which is clearly positive.

It is easy to see that the cost excess is below $\frac{D_1}{D_1+D_2}\Big[(\max_{i\,\in\,\mathbf{S}} h_i)D_2 - \min_{i\,\in\,\mathbf{S}} f_i\Big]$. Indeed:

$$\begin{aligned}
\frac{D_1}{D_1+D_2}\Big[(\max_{i\,\in\,\mathbf{S}} h_i)D_2 - \min_{i\,\in\,\mathbf{S}} f_i\Big] &= \frac{D-\varepsilon}{D-\varepsilon+D}\,0.02\,D \\
&= \frac{2\,D}{2\,D-\varepsilon}\,0.01\,(D-\varepsilon) \\
&\geq 0.01\,(D-\varepsilon) \\
&= J^{\mathrm{M}}-J\,.
\end{aligned}$$

Moreover, if $\varepsilon \downarrow 0$ then $J^{\mathrm{M}}-J \downarrow 0.01\,D = \frac{1}{2}\Big[(\max_{i\,\in\,\mathbf{S}} h_i)D_2 - \min_{i\,\in\,\mathbf{S}} f_i\Big]$. $\square$

In the following example, we illustrate the need of $\frac{1}{2}$ in the bound in Theorem 4.1, by showing that there exists a class of problem instances such that the cost excess satisfies

$$\max\left\{\max_{i\,\in\,\mathbf{S}} f_i,\ \frac{D_1}{D_1+D_2}\Big[(\max_{i\,\in\,\mathbf{S}} h_i)D_2 - \min_{i\,\in\,\mathbf{S}} f_i\Big]\right\} \;<\; J^{\mathrm{M}}-J \;\leq\; \frac{1}{2}\Big[(\max_{i\,\in\,\mathbf{S}} h_i)D_2 - \min_{i\,\in\,\mathbf{S}} f_i\Big]. \tag{4}$$

Obviously, $\frac{D_1}{D_1+D_2} < \frac{1}{2}$, and thus in such a problem instance $D_1 < D_2$. Again, the problem instance is in line with that one in Example 3.1, though the cost figures are slightly different, the setup costs are discriminatory and $D_1 < D_2$.

Example 4.3 Let the demands be $(D,\,2D)$, where $D \in [225, 250]$. There are two suppliers, each of which faces a unit holding cost of 0.022. The other supplier data are shown in the table below.

i	f_i	p_i	b_{i1}	b_{i2}
1	1	0.100	$3\,D$	0
2	1.95	0.111	D	$2\,D$

Let us first look at the Standard auction. The timing of production is similar to that in Example 3.1, and

$$\begin{aligned}
C_1 &= 1 + 0.10\,(D{+}2\,D) + 0.022\,(2\,D) &&= 1 + 0.344\,D, \\
C_2 &= 1.95 + 0.111\,(D) + 1.95 + 0.111\,(2\,D) &&= 3.9 + 0.333\,D.
\end{aligned}$$

Since $D \leq 250$, the winner of this auction is supplier 1 with:

$$C_{\mathbf{S}} = 1 + 0.344\,D \quad [w{=}1].$$

In the Multiple Winner auction, it is easy to see that policy (ii) is dominated by policy (i), as both suppliers have the same unit inventory holding cost and supplier 2 is more expensive in terms of setup as well as unit production costs. Therefore:

$$
\begin{aligned}
C^{\mathrm{M}}_{\mathbf{S}} &= \min\left\{\min_i\{f_i + p_i(D_1{+}D_2) + h_i(D_2)\},\ \min_{i,i'}\{(f_i + p_i D_1) + (f_{i'} + p_{i'} D_2)\}\right\} \\
&= \min\left\{1 + 0.344\, D\ \ [w_1{=}w_2{=}1],\ \min_{i,i'}\{(f_i + p_i D_1) + (f_{i'} + p_{i'} D_2)\}\right\}
\end{aligned}
$$

Using again that supplier 2 is more expensive in terms of setup and unit production costs

$$
\begin{aligned}
&= \min\,\{1 + 0.344\, D\ \ [w_1{=}w_2{=}1],\ \ (1 + 0.10\,(D)) + (1.95 + 0.111\,(2\,D))\ \ [w_1{=}1,\ w_2{=}2]\} \\
&= \min\,\{1 + 0.344\, D,\ \ 2.95 + 0.322\, D\} \\
&= 2.95 + 0.322\, D \quad [w_1{=}1,\ w_2{=}2],
\end{aligned}
$$

where the last equality follows since $D \geq 225$. Thus, the lowest cost policy is policy (iii), with supplier 1 producing D_1 and supplier 2 producing D_2 in period 2.

Since there are only two suppliers and the winner of the Standard auction is supplier 1, the cost excess is given by $C_1 - C^{\mathrm{M}}_{\mathbf{S}}$:

$$
\begin{aligned}
J^{\mathrm{M}} - J &= C_1 - C^{\mathrm{M}}_{\mathbf{S}} \\
&= (1 + 0.344\, D) - (2.95 + 0.322\, D) \\
&= 0.022\, D - 1.95,
\end{aligned}
$$

which is positive since $D \geq 225$.

Now we will show that the cost excess satisfies (4) showing the need of $\max\left\{\frac{D_1}{D_1+D_2}, \frac{1}{2}\right\}$ in the cost excess bound. Clearly, we only need to show the inequality of the left hand side. Indeed:

$$
\begin{aligned}
\max\left\{\max_{i\,\in\,\mathbf{S}} f_i,\ \frac{D_1}{D_1{+}D_2}\left[(\max_{i\,\in\,\mathbf{S}} h_i) D_2 - \min_{i\,\in\,\mathbf{S}} f_i\right]\right\} &= \max\left\{1.95,\ \frac{D}{D{+}2\,D}\,(0.022\,(2\,D) - 1)\right\} \\
&= \max\left\{1.95,\ \frac{1}{3}\,(0.044\, D - 1)\right\}
\end{aligned}
$$

Since $D \geq 225$, the maximum is achieved at the second term and

$$
\begin{aligned}
&< 0.022\, D - 1.95 \\
&= J^{\mathrm{M}} - J,
\end{aligned}
$$

and the desired inequality follows. $\square$

The following result is a corollary to Theorem 4.1.

Corollary 4.4 *The cost excess is bounded from above as follows:*

$$J^{\mathrm{M}} - J \quad \leq \quad \max \left\{ \max_{i \in \mathbf{S}} f_i, \; \max\left\{ \frac{D_1}{D_1 + D_2}, \frac{1}{2} \right\} \left[H\, D_2 - \min_{i \in \mathbf{S}} f_i \right] \right\}.$$

4.2 Tightness of the cost excess bound

In this section we show that the worst case bound found in Theorem 4.1 cannot be improved. We will do this by finding a class of problem instances for which the cost excess is indeed equal to (3). Since the bound is the maximum of two terms, we will prove the result in two steps.

In the following proposition we show that the first part of the bound in Theorem 4.1 is tight.

Proposition 4.5 *There exists a class of problem instances for which the cost excess is equal to*

$$J^{\mathrm{M}} - J \quad = \quad \max_{i \in \mathbf{S}} f_i.$$

In the following proposition we show that the second part of the bound in Theorem 4.1 is tight.

Proposition 4.6 *There exists a class of problem instances for which the cost excess is equal to*

$$J^{\mathrm{M}} - J \quad = \quad \max\left\{ \frac{D_1}{D_1 + D_2}, \frac{1}{2} \right\} \left[H\, D_2 - \min_{i \in \mathbf{S}} f_i \right].$$

The following result is a corollary to Propositions 4.5 and 4.6.

Corollary 4.7 *The cost excess bound given in Theorem 4.1 is tight.*

4.3 Sharpening the cost excess bound

We consider in this section two scenarios that can arise when the buyer announces his demands close to the beginning of his demand horizon, and thus the suppliers have already made some commitments with other buyers. In the first scenario, each supplier is already committed to producing in both periods but may have some spare production capacity in one or both periods. In the second scenario, each supplier has available capacity in only one period.

In the first scenario, where each supplier is already committed to production in both periods, this means that the production setup costs for each supplier are already sunk and thus can be taken to be zero. We will in fact prove the proposition in a slightly more general form where the setup costs can be nonzero but equal across suppliers.

If the setup costs are supplier-independent, the bound on the cost excess in Theorem 4.1 can be improved, where $\max\left\{\frac{D_1}{D_1+D_2}, \frac{1}{2}\right\}$ is replaced by $\frac{D_1}{D_1+D_2}$.

Proposition 4.8 *If the setup costs are supplier-independent and equal to f, the cost excess is bounded from above as follows:*

$$J^{\mathrm{M}} - J \;\le\; \max\left\{f,\ \frac{D_1}{D_1+D_2}\Big[HD_2 - f\Big]\right\}.$$

Moreover, this bound is tight.

If, indeed, $f = 0$, we then have the following two corollaries.

Corollary 4.9 *If the setup costs are equal to* 0*, the cost excess is bounded from above as follows:*

$$J^{\mathrm{M}} - J \;\le\; \frac{D_1 D_2}{D_1+D_2}\, H.$$

Moreover, this bound is tight.

Corollary 4.10 *If the setup costs are equal to* 0*, the cost excess is bounded from above as follows:*

$$J^{\mathrm{M}} - J \;\le\; \frac{1}{2}\max\{D_1, D_2\}\, H.$$

Moreover, when the demands are equal, this bound is tight.

If each supplier has capacity available to him in only one period, the bound on the cost excess in Theorem 4.1 can be improved further.

Proposition 4.11 *If each supplier has capacity available to him in only one period, the cost excess is bounded from above as follows:*

$$J^{\mathrm{M}} - J \;\le\; \max\left\{\max_{i\,\in\,\mathbf{S}} f_i,\ \frac{D_1}{D_1+D_2}\Big[H\,D_2 - \min_{i\,\in\,\mathbf{S}} f_i\Big]\right\}.$$

Moreover, this bound is tight.

5 A monotonicity condition

We consider now a monotonicity condition. This condition is satisfied, for instance, when the holding costs are a fixed percentage of the unit production costs, see Silver et al. (1998):

Monotonicity Condition. For each i, $i' \in \mathbf{S}$, if $p_i \leq p_{i'}$ *then* $h_i \leq h_{i'}$.

Under this monotonicity condition, we derive below a bound on the cost excess when capacity is only available in one period, and show its tightness. These results are derived for the general T-period case. As a corollary, and when the number of periods is equal to 2, we show that the bound in Proposition 4.11 can be sharpened.

Proposition 5.1 *In the T-period case, if each supplier has capacity in only one period and the monotonicity condition is satisfied, the cost excess is bounded from above as follows:*

$$J^{\mathrm{M}} - J \quad \leq \quad (W-1) \max_{i \in \mathbf{S}} f_i.$$

The following example illustrates the necessity of the monotonicity condition.

Example 5.2 Consider the 2-period case. Let the demand in each period be D. There are two suppliers, each of which faces a setup cost of 0 in each period and no capacity in period 2. The other supplier data are shown in the table below where $0 < \frac{\delta}{2} < \varepsilon < \delta < h$.

i	f_i	p_i	h_i	b_{i1}	b_{i2}
1	0	p	h	$2\,D$	0
2	0	$p+\varepsilon$	$h-\delta$	$2\,D$	0

Clearly, the monotonicity condition is violated since $p_1 < p_2$ but $h_1 > h_2$.

The bound given in Proposition 5.1 is clearly equal to zero, as all the setup costs are equal to zero. In the following, we will show that the cost excess for this problem instance is positive and thus, the monotonicity condition is a necessary requirement in that proposition.

Let us first look at the Standard auction. Due to the lack of capacity in period 2, both suppliers need to carry the second demand in inventory and

$$\begin{aligned} C_1 &= 0 + p\,(D+D) + h\,(D) &&= (2\,p+h)D \\ C_2 &= 0 + (p+\varepsilon)\,(D+D) + (h-\delta)\,(D) &&= (2\,p+h)D + (2\,\varepsilon-\delta)\,D. \end{aligned}$$

Since $\varepsilon > \frac{\delta}{2}$, the winner of the Standard auction is supplier 1 with

$$C_{\mathbf{S}} \;=\; (2\,p{+}h)D \quad [w{=}1].$$

Now let us look at the Multiple Winner auction. Because the setup costs are equal to zero, in the Multiple Winner auction each demand will be allocated to the supplier with the lowest total unit cost. Since in our problem instance $p_1 < p_2 < p_2{+}h_2 < p_1{+}h_1$, the Multiple Winner auction will have two winners. Indeed:

$$\begin{aligned}
C_{\mathbf{S}}^{\mathrm{M}} &= \min\{p_1, p_2\}\, D_1 + \min\{p_1{+}h_1, p_2{+}h_2\}\, D_2 \\
&= p_1\, D + (p_2{+}h_2)\, D \\
&= p\, D + (p + \varepsilon + h - \delta)\, D \\
&= (2\,p{+}h)D - (\delta{-}\varepsilon)\, D \;\; [w_1{=}1,\; w_2{=}2].
\end{aligned}$$

Since there are only two suppliers and the winner of the Standard auction is supplier 1, the cost excess is given by $C_1 {-} C_{\mathbf{S}}^{\mathrm{M}}$, thus:

$$\begin{aligned}
J^{\mathrm{M}} {-} J &= C_1 - C_{\mathbf{S}}^{\mathrm{M}} \\
&= (\delta - \varepsilon)\, D \\
&> 0
\end{aligned}$$

since $\varepsilon < \delta$. $\square$

The following result is a corollary to Proposition 5.1, where the number of winners in the Multiple Winner auction is bounded by $\min\{S, T\}$.

Corollary 5.3 *In the T-period case, if each supplier has capacity in only one period and the monotonicity condition is satisfied, the cost excess is bounded from above as follows:*

$$J^{\mathrm{M}} {-} J \;\leq\; (\min\{S, T\}{-}1)\, \max_{i \,\in\, \mathbf{S}} f_i.$$

Thus, the cost to the buyer of the Multiple Winner auction cannot exceed that of the Standard auction by more than the following quantity: The smaller of the number of suppliers and the number of periods minus one, times the largest setup cost.

The following result shows the tightness of the bound in Corollary 5.3.

Proposition 5.4 *The bound given in Proposition 5.1 is tight.*

The following result is a corollary to Propositions 5.1 and 5.4, when $T = 2$.

Corollary 5.5 *In the 2-period case, if each supplier has capacity in only one period and the monotonicity condition is satisfied, the cost excess is bounded from above as follows:*

$$J^{\mathrm{M}} - J \quad \leq \quad \max_{i \in \mathbf{S}} f_i.$$

Moreover, this bound is tight.

6 Conclusions

We have considered the case of a firm, called the buyer, who satisfies his demand over two periods, by either auctioning his demand profile to the least costly supplier (the "Standard auction") via the Vickrey auction, or to the least costly set of suppliers, including the possibility of package bids ("the Multiple Winner auction"), via the VCG auction. The Multiple Winner auction can in fact result in a higher cost to the buyer than the Standard auction. We provide a bound on how much the cost to the auctioneer of the Multiple Winner auction can exceed the cost of the Standard auction. Further, we show via a class of problem instances that this bound is sharp. Some of the results of the paper will hold under more general cost functions, although the analysis becomes complex.

The bound described above is the greater of two quantities. The first quantity is the maximum possible supplier setup cost. The second quantity is the maximum cost incurred by any supplier in holding the second period demand over from the first period, minus the minimum possible setup cost, where this difference is multiplied by the first period demand over the sum of the demands or by 1/2, whichever is larger. This bound is based on information the buyer is likely to know to a reasonable degree. Of course, the buyer knows his own demands. Further, he is likely to understand the technology of his suppliers to the extent that he would be able to estimate their setup costs. The supplier holding costs can be estimated by the buyer's own holding cost which, by a standard supply chain assumption, will dominate the suppliers' holding costs.

Our further results are derived for two scenarios that can arise when the buyer announces his demands close to the beginning of his demand horizon, and thus the suppliers may have already made commitments with other buyers. In the first scenario, the setup costs are supplier-independent. We find that we can improve the earlier bound. In the second scenario, each supplier has capacity available only in a single period. We find that we can improve the earlier bound in this scenario as well. If, in addition, a mild monotonicity assumption holds, we can further improve the bound, and show that this bound holds not only in the 2-period case, but in fact holds in the general T-period case.

For each of these improvements, we show that the bound is sharp.

One may ask what factors drive the results. We saw in the introduction an example with four suppliers, where two suppliers have essentially unlimited capacity in both periods, while the other two suppliers have capacity only in period 1. In this example, the two capacity-constrained suppliers have already sunk their setup costs in both periods—and can therefore be assumed to be zero—and their remaining capacity is only sufficient to satisfy one period demand for each of them. We found here that the revenue deficiency equals the setup cost. However, even if the setup costs do not play a role, e.g., they are sunk, revenue deficiency may still occur. In Section 4, we considered the case of two suppliers, where the first supplier has essentially unlimited capacity in the first period and no remaining capacity in the second period, while the second supplier has remaining capacity in each period that is sufficient to satisfy one period of demand. In this example, the unit production cost of the second supplier exceeds the unit production cost of the first supplier by half of the unit inventory holding cost. This scenario results in a revenue deficiency equal to half the cost of holding the second period demand over from the first period.

Finally, Ausubel and Milgrom (2006) point out that, since truthful reporting is a dominant strategy under the VCG mechanism, the suppliers have no incentive to spend resources learning about competitor values' or strategies'. Our results bring in a new dimension from the point of view of the buyer. Specifically, the buyer would have a clear incentive to spend resources learning about the production capacities of his suppliers, as well as whether they engage in *resource pooling*. Chod and Rudi (2006) describe how, when making capacity, inventory, and production decisions, firms are typically uncertain about future market conditions, and that the opportunity cost associated with the quantity decisions made under uncertainty can often be mitigated by various resource pooling strategies, i.e., arrangements in which independent firms trade or subcontract capacity. Chod and Rudi (2006) point out that this practice is widespread in the telecommunications, pharmaceutical, and electronics manufacturing industries. The significance of this in our context is that, with resource pooling the suppliers could be considered in some circumstances to be virtually uncapacitated. Thus, a buyer sourcing via procurement auctions would have a clear incentive to spend resources learning to what extent this practice is common among his suppliers.

Appendix

The proofs of the results of the paper are given at the end of the appendix. In the following we establish an upper bound on the cost excess when either w or $w\_$ is not a winner of the Multiple Winner auction. This cost excess bound is shown for a general T. Before we show this result, we require several technical results. The following lemma is a trivial observation about a scenario for which the cost excess clearly must be nonpositive.

Lemma A.1 *In the T-period case, if the Multiple Winner auction has only one winner, then $J^{\mathrm{M}} \leq J$.*

Proof: If the Multiple Winner auction has only one winner, then clearly this should be the winner of the Standard auction, i.e., $\mathbf{W} = \{w_1\} = \{w\}$. Now the costs excess is clearly nonpositive. Indeed:

$$J^{\mathrm{M}} - J \;=\; C^{\mathrm{M}}_{\mathbf{S}\setminus\{w\}} - C_{\mathbf{S}\setminus\{w\}} \;\leq\; 0.$$

$\square$

In the following we analyze the case in which the Multiple Winner auction has at least two winners. In the following we introduce some notation that will be used throughout this appendix. Let $\mathbf{T}' \subseteq \mathbf{T}$ be a subset of period demands. Since w is the winner of the Standard auction, it has enough capacity to produce $\mathbf{T}'$. Let $C_w(\mathbf{T}')$ denote the total cost to w in C_w incurred when delivering the demands in $\mathbf{T}'$. Notice that $C_w(\mathbf{T}')$ may include costs incurred in periods outside $\mathbf{T}'$. Even further, $C_w(\mathbf{T}')$ may include setup costs shared with period demands outside $\mathbf{T}'$. Similarly, we can define $C_{w\_}(\mathbf{T}')$. Finally, let $\mathbf{T}_i$ denote the set of demands won by supplier $i \in \mathbf{W}$ in the Multiple Winner auction, where $C^{\mathrm{M}}_{\mathbf{S},i}$ denotes the cost to i of supplying its allocation $\mathbf{T}_i$.

In the following, we give a useful expression of the cost excess $J^{\mathrm{M}} - J$.

Lemma A.2 *In the T-period case, the expression of $J^{\mathrm{M}} - J$ is equal to*

$$J^{\mathrm{M}} - J \;=\; \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i})] - C_{w\_}. \tag{5}$$

Proof: From equation (2) and equation (1), we have that:

$$\begin{aligned} J^{\mathrm{M}} - J \;&=\; C^{\mathrm{M}}_{\mathbf{S}} + \sum_{i \in \mathbf{W}} (C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - C^{\mathrm{M}}_{\mathbf{S}}) - C_{w\_} \\ &=\; \sum_{i \in \mathbf{W}} C^{\mathrm{M}}_{\mathbf{S},i} + \sum_{i \in \mathbf{W}} (C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - C^{\mathrm{M}}_{\mathbf{S}}) - C_{w\_} \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S},i} + (C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - C^{\mathrm{M}}_{\mathbf{S}})] - C_{w\_} \\
&= \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i})] - C_{w\_}.
\end{aligned}
$$

□

The following lemma gives an upper bound on the costs $C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}}$, $i \in \mathbf{W}$, when either w or $w\_ \notin \mathbf{W}$.

Lemma A.3 *In the T-period case, the following cost relationships hold:*

$$
\begin{aligned}
C_{w\_}(\mathbf{T}_i) + (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i}) &\geq C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} \qquad i \in \mathbf{W},\ w\_ \notin \mathbf{W} \\
C_{w}(\mathbf{T}_i) + (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i}) &\geq C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} \qquad i \in \mathbf{W},\ w \notin \mathbf{W}.
\end{aligned}
$$

Proof: We will establish these inequalities by finding an allocation of the period demands in the Multiple Winner auction in the absence of supplier $i \in \mathbf{W}$, based on the optimal allocation of the period demands in the Multiple Winner auction.

Let us first assume that $i \in \mathbf{W}$ with $w\_ \notin \mathbf{W}$. This means that $i \neq w\_$. The right hand side of the inequality corresponding to this case is by definition the lowest cost of supplying all the demands in the Multiple Winner auction when supplier i is not present. In the absence of i, all the demands allocated to him, $\mathbf{T}_i$, can be assigned to $w\_$. Since $w\_ \notin \mathbf{W}$, $\mathbf{T}_i$ can be produced using any production plan, as long as it does not violate the capacity constraints faced by $w\_$. We will assume that the production timing of the period demands in $\mathbf{T}_i$ will be the same as the one used by $w\_$ when being responsible for all period demands. Therefore the costs incurred by $w\_$ will be by definition $C_{w\_}(\mathbf{T}_i)$. Demand in the remaining periods is supplied under the optimal allocation $C^{\mathrm{M}}_{\mathbf{S}}$, at a cost of $C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i}$, and the desired inequality follows.

The proof is similar for $i \in \mathbf{W}$ with $w \notin \mathbf{W}$. We just need to assign the demands in $\mathbf{T}_i$ to w. □

The following lemma bounds the cost excess when either w or $w\_$ is not a winner in the Multiple Winner auction.

Lemma A.4 *In the T-period case, if either w or $w\_$ is not a winner in the Multiple Winner auction*

$$
J^{\mathrm{M}} - J \quad \leq \quad (W-1)\,(\max_{i \in \mathbf{S}} f_i).
$$

Proof: Suppose $w\_ \notin \mathbf{W}$, i.e. $\mathbf{T}_{w\_} = \emptyset$. By Lemma A.2, the cost excess can be written as:

$$
\begin{aligned}
J^{\mathrm{M}} - J &= \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i})] - C_{w\_} \\
&= \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i}) + (-C_{w\_}(\mathbf{T}_i) + C_{w\_}(\mathbf{T}_i))] - C_{w\_} \\
&= \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i}) - C_{w\_}(\mathbf{T}_i)] + \sum_{i \in \mathbf{W}} C_{w\_}(\mathbf{T}_i) - C_{w\_}, \text{ by Lemma A.3} \\
&\leq 0 + \sum_{i \in \mathbf{W}} C_{w\_}(\mathbf{T}_i) - C_{w\_}.
\end{aligned}
$$

Thus, it is enough to show that $\sum_{i \in \mathbf{W}} C_{w\_}(\mathbf{T}_i) - C_{w\_}$ is bounded by $(W-1)\max_{i \in \mathbf{S}} f_i$. Notice that $\{\mathbf{T}_i\}_{i \in \mathbf{W}}$ is a partition of the time horizon $\mathbf{T}$. Second, and for all $t \in \mathbf{T}$, the timing of production of period demand D_t in both $\sum_{i \in \mathbf{W}} C_{w\_}(\mathbf{T}_i)$ and $C_{w\_}$ is the same, by definition of $C_{w\_}(\mathbf{T}_i)$. Therefore, the total variable production and holding costs in both $\sum_{i \in \mathbf{W}} C_{w\_}(\mathbf{T}_i)$ and $C_{w\_}$ are the same. With respect to the setup costs, it is clear that $\sum_{i \in \mathbf{W}} C_{w\_}(\mathbf{T}_i)$ incurs at least the same number of setup costs as $C_{w\_}$. However, we may need to incur an extra setup cost to start production in each subset $\mathbf{T}_i$, such that $D_1 \notin \mathbf{T}_i$. Because there are $W-1$ of these subsets, $\sum_{i \in \mathbf{W}} C_{w\_}(\mathbf{T}_i) - C_{w\_} \leq (W-1) f_{w\_}$, and the desired bound follows.

A similar bound can be derived if $w \notin \mathbf{W}$. In this case, the cost excess can be bounded by

$$
\begin{aligned}
J^{\mathrm{M}} - J &= \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i})] - C_{w\_} \\
&= \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i})] - C_w + C_w - C_{w\_} \\
&\leq \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i})] - C_w \\
&= \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i}) + (-C_w(\mathbf{T}_i) + C_w(\mathbf{T}_i))] - C_w \\
&= \sum_{i \in \mathbf{W}} [C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - (C^{\mathrm{M}}_{\mathbf{S}} - C^{\mathrm{M}}_{\mathbf{S},i}) - C_w(\mathbf{T}_i)] + (\sum_{i \in \mathbf{W}} C_w(\mathbf{T}_i) - C_w).
\end{aligned}
$$

Now the proof follows in a similar fashion as above. $\square$

Proof of Theorem 4.1

The bound follows when the Multiple Winner auction has exactly one winner, see Lemma A.1, or when either w or $w\_$ is not a winner in the Multiple Winner auction, see Lemma A.4. Therefore, in the rest of the proof we will assume that $\mathbf{W} = \{w, w\_\}$, with $w \neq w\_$. In this case, the cost excess can be bounded by:

$$\begin{aligned} J^{\mathrm{M}} - J &= C^{\mathrm{M}}_{\mathbf{S}\setminus\{w\}} + C^{\mathrm{M}}_{\mathbf{S}\setminus\{w\_\}} - C^{\mathrm{M}}_{\mathbf{S}} - C_{w\_} \\ &\le C^{\mathrm{M}}_{\{w\_\}} + C^{\mathrm{M}}_{\{w\}} - C^{\mathrm{M}}_{\mathbf{S}} - C_{w\_} \\ &= C_{w\_} + C_w - C^{\mathrm{M}}_{\mathbf{S}} - C_{w\_} \\ &= C_w - C^{\mathrm{M}}_{\mathbf{S}} \end{aligned}$$

where this last term corresponds to the cost excess when the set of suppliers taking part in the bidding process is equal to $\{w, w\_\}$. Therefore, in the following we will assume that $\mathbf{S} = \{w, w\_\}$.

It remains to prove that

$$C_w - C^{\mathrm{M}}_{\mathbf{S}} \le \max\{\max_{i \in \mathbf{S}} f_i, \max\left\{\frac{D_1}{D_1+D_2}, \frac{1}{2}\right\}\left[(\max_{i \in \mathbf{S}} h_i) D_2 - \min_{i \in \mathbf{S}} f_i\right]\}. \tag{6}$$

We will prove (6) by distinguishing cases defined by the capacity faced by w and $w\_$ and the production timing of both demands in C_w and $C_{w\_}$. In particular, we will distinguish the following three possibilities for each of the two suppliers, leaving in total nine cases to be analyzed: (i) capacity is only available in period 1 and therefore both demands are produced in period 1, (ii) capacity in period 1 is not enough to produce both demands and therefore each demand is produced in its own period, and (iii) there is enough capacity in each period and both demands are produced in period 1. We may observe that there is one remaining possibility, i.e., there is enough capacity in each period and each demand is produced in its own period. In this case, if this supplier is allocated demand D_2, this demand will be produced in period 2 as this is feasible and at least as cheap as producing it in period 1. This means that the capacity in period 1 will never be used to produce D_2. Thus, it will be enough to consider possibility (ii).

Before we analyze each case, we derive a lower bound on the difference in unit production costs, $p_w - p_{w\_}$, based on the inequality $C_{w\_} \ge C_w$. This lower bound will be used when bounding the cost excess. We have that:

$$\begin{aligned} C_{w\_} &\ge C_w \\ f_{w\_} + p_{w\_}(D_1+D_2) + \Pi_{w\_} &\ge f_w + p_w(D_1+D_2) + \Pi_w \end{aligned}$$

$$
\begin{aligned}
f_{w\_} - f_w + \Pi_{w\_} - \Pi_w &\geq (p_w - p_{w\_})\,(D_1{+}D_2) \\
\frac{1}{D_1{+}D_2}\,[f_{w\_} - f_w + \Pi_{w\_} - \Pi_w] &\geq p_w - p_{w\_}
\end{aligned} \tag{7}
$$

where $\Pi_s \in \{f_s, h_s\, D_2\}$ and $s = w, w\_$.

1. Both w and $w\_$ each have capacity only in period 1.

   This means that in C_w, $C_{w\_}$ and $C^{\mathrm{M}}_{\mathbf{S}}$, D_2 will be produced in period 1, and all three solutions incur inventory holding costs when delivering D_2. Moreover, in $C^{\mathrm{M}}_{\mathbf{S}}$, D_1 will be allocated to the supplier with the cheapest unit production costs, i.e., $\arg\min\{p_w, p_{w\_}\}$, while D_2 will be allocated to the supplier with the cheapest unit production costs plus unit inventory holding costs, i.e., $\arg\min\{p_w{+}h_w, p_{w\_}{+}h_{w\_}\}$. Since the Multiple Winner auction has two winners, these two suppliers should be different, i.e.,

$$
\arg\min\{p_w, p_{w\_}\} \neq \arg\min\{p_w + h_w, p_{w\_} + h_{w\_}\}.
$$

   This means that either

$$
p_w \leq p_{w\_} \leq p_{w\_} + h_{w\_} \leq p_w + h_w \qquad \text{or} \tag{8}
$$
$$
p_{w\_} \leq p_w \leq p_w + h_w \leq p_{w\_} + h_{w\_}, \tag{9}
$$

   where in both cases at least one inequality should be strict. We will analyze these two cases separately.

   (a) Suppose that (8) holds, where at least one of these inequalities is strict. This means that in $C^{\mathrm{M}}_{\mathbf{S}}$, D_1 is assigned to w and D_2 is assigned to $w\_$.
   The cost excess now can be bounded using (7) where $\Pi_s = h_s\, D_2$:

$$
\begin{aligned}
C_w - C^{\mathrm{M}}_{\mathbf{S}} &= (p_w\, D_2 + h_w\, D_2) - (f_{w\_} + p_{w\_} D_2 + h_{w\_}\, D_2) \\
&= (p_w - p_{w\_})D_2 + h_w\, D_2 - h_{w\_}\, D_2 - f_{w\_} \\
&\leq \frac{D_2}{D_1{+}D_2}[f_{w\_} - f_w + h_{w\_}\, D_2 - h_w\, D_2] + h_w\, D_2 - h_{w\_}\, D_2 - f_{w\_} \\
&= (1 - \frac{D_2}{D_1{+}D_2})[(h_w - h_{w\_})D_2 - f_{w\_}] - \frac{D_2}{D_1{+}D_2} f_w \\
&\leq \frac{D_1}{D_1{+}D_2}[h_w\, D_2 - f_{w\_}].
\end{aligned}
$$

   (b) Suppose that (9) holds, where at least one of these inequalities is strict. This means that in $C^{\mathrm{M}}_{\mathbf{S}}$, D_1 is assigned to $w\_$ and D_2 is assigned to w.

The cost excess now can be again bounded using (7):

$$\begin{aligned}
C_w - C_{\mathbf{S}}^{\mathrm{M}} &= p_w\, D_1 - (f_{w_} + p_{w_} D_1) \\
&= (p_w - p_{w_})D_1 - f_{w_} \\
&\leq \frac{D_1}{D_1+D_2}[f_{w_} - f_w - (h_w - h_{w_})D_2] - f_{w_} \\
&= \frac{D_1}{D_1+D_2}[(h_{w_} - h_w)D_2 - f_w] + (\frac{D_1}{D_1+D_2} - 1)f_{w_} \\
&\leq \frac{D_1}{D_1+D_2}[h_{w_}\, D_2 - f_w].
\end{aligned}$$

Thus, in both cases (6) follows.

2. Both w and $w\_$ have capacity in both periods but the capacity in period 1 is not enough to produce both demands.

   This means that in C_w, $C_{w\_}$ and $C_{\mathbf{S}}^{\mathrm{M}}$, D_2 will be produced in period 2 and therefore no inventory holding costs are incurred. In order to prove the result, we will distinguish two cases for the unit production costs.

   Suppose that $p_w \leq p_{w\_}$. Because in C_w both demands are allocated to w while in $C_{\mathbf{S}}^{\mathrm{M}}$ one will be allocated instead to $w\_$, in terms of total production costs C_w is as cheap as $C_{\mathbf{S}}^{\mathrm{M}}$. Thus, when bounding the cost excess $C_w - C_{\mathbf{S}}^{\mathrm{M}}$ we focus on the setup costs, and

   $$C_w - C_{\mathbf{S}}^{\mathrm{M}} \leq 2\, f_w - (f_w + f_{w\_}) = f_w - f_{w\_}.$$

   If $p_w > p_{w\_}$, by noting that the cost excess is bounded by $C_{w\_} - C_{\mathbf{S}}^{\mathrm{M}}$, we have that

   $$C_w - C_{\mathbf{S}}^{\mathrm{M}} \leq C_{w\_} - C_{\mathbf{S}}^{\mathrm{M}} \leq 2\, f_{w\_} - (f_w + f_{w\_}) = f_{w\_} - f_w.$$

   Thus, in both cases the cost excess is bounded by $\max_{i\,\in\,\mathbf{S}} f_i$ and (6) follows.

3. Both w and $w\_$ have enough capacity in both periods and produce both demands in period 1.

   This means that in $C_{\mathbf{S}}^{\mathrm{M}}$, D_2 will be produced in period 2. Thus, $C_{\mathbf{S}}^{\mathrm{M}}$ is as in Case 2. With respect to C_w and $C_{w\_}$, both suppliers produce both demands in period 1, but there is an alternative solution, at least as expensive, in which each demand is produced in its own period as in Case 2. Now it is easy to see that the cost excess bound derived in Case 2 applies here too.

4. Supplier w has only capacity in period 1, while $w\_$ has capacity in both periods but the capacity in period 1 is not enough to produce both demands.

   We will distinguish two cases depending on the supplier in charge of demand D_2 in $C^{\mathrm{M}}_{\mathbf{S}}$.

   (a) If D_2 is allocated to w in $C^{\mathrm{M}}_{\mathbf{S}}$, D_2 will be produced in period 1 since w has only capacity in period 1. This means that both C_w and $C^{\mathrm{M}}_{\mathbf{S}}$ incur inventory holding costs.

   The cost excess can be bounded using (7) where $\Pi_w = h_w\, D_2$ and $\Pi_{w\_} = f_{w\_}$:

$$\begin{aligned} C_w - C^{\mathrm{M}}_{\mathbf{S}} &= p_w\, D_1 - (f_{w\_} + p_{w\_} D_1) \\ &= (p_w - p_{w\_})\, D_1 - f_{w\_} \\ &\leq \frac{D_1}{D_1{+}D_2}\,[2\, f_{w\_} - f_w - h_w\, D_2] - f_{w\_} \\ &\leq \frac{2\, D_1}{D_1{+}D_2}\, f_{w\_} - f_{w\_} \\ &= \frac{D_1 - D_2}{D_1{+}D_2}\, f_{w\_} \\ &\leq f_{w\_}, \end{aligned}$$

   and (6) follows.

   (b) If D_2 is assigned to $w\_$ in $C^{\mathrm{M}}_{\mathbf{S}}$, D_2 will be produced in period 2, and neither $C_{w\_}$ or $C^{\mathrm{M}}_{\mathbf{S}}$ incur inventory holding costs. If $p_w \geq p_{w\_}$, as in Case 2, the cost excess is bounded by

$$C_w - C^{\mathrm{M}}_{\mathbf{S}} \;\leq\; f_{w\_} - f_w$$

   and inequality (6) follows.

   Now, suppose that $p_{w\_} > p_w$.

   i. If $D_2 > D_1$, from inequality $C_{w\_} \geq C_w$ we have that

$$\begin{aligned} C_{w\_} &\geq C_w \\ 2\, f_{w\_} + p_{w\_}(D_1{+}D_2) &\geq f_w + p_w(D_1{+}D_2) + h_w\, D_2 \\ 2\,(f_{w\_} - f_w) + (p_{w\_} - p_w)(D_1{+}D_2) &\geq -f_w + h_w\, D_2 \\ 2\,(f_{w\_} - f_w) + 2\,(p_{w\_} - p_w)D_2 &\geq -f_w + h_w\, D_2 \quad \text{Since } D_2 > D_1 : \\ (f_{w\_} - f_w) + (p_{w\_} - p_w)D_2 &\geq \frac{1}{2}\,(-f_w + h_w\, D_2) \end{aligned}$$

$$(f_{w\_} - f_w) + \frac{1}{2}\,(f_w - h_w\, D_2) \quad \geq \quad (p_w - p_{w\_})\, D_2. \tag{10}$$

The cost excess now can be bounded using (10)

$$\begin{aligned}
C_w - C_{\mathbf{S}}^{\mathrm{M}} &= (p_w\, D_2 + h_w\, D_2) - (f_{w_} + p_{w_}\, D_2) \\
&= (p_w - p_{w_})\, D_2 + h_w\, D_2 - f_{w_} \\
&\leq (f_{w_} - f_w) + \frac{1}{2}\,(f_w - h_w\, D_2) + h_w\, D_2 - f_{w_} \\
&= \frac{1}{2}\,(h_w\, D_2 - f_w),
\end{aligned}$$

and (6) follows.

ii. Otherwise, $D_2 \leq D_1$. Using (7):

$$\begin{aligned}
C_w - C_{\mathbf{S}}^{\mathrm{M}} &= (p_w - p_{w_})\, D_2 + h_w\, D_2 - f_{w_} \\
&\leq \frac{1}{D_1 + D_2}\,\{2\, f_{w_} - f_w - h_w\, D_2\}\, D_2 + h_w\, D_2 - f_{w_} \\
&= (1 - \frac{D_2}{D_1 + D_2})\, h_w\, D_2 - \frac{D_2}{D_1 + D_2}\,\{f_w - 2\, f_{w_}\} - f_{w_} \\
&= \frac{D_1}{D_1 + D_2}\, h_w\, D_2 - \frac{1}{D_1 + D_2}\,\{D_2\, f_w + (D_1 - D_2)\, f_{w_}\} \\
&= \frac{D_1}{D_1 + D_2}\Big[h_w\, D_2 - \{\frac{D_2}{D_1}\, f_w + (1 - \frac{D_2}{D_1})\, f_{w_}\}\Big] \\
&= \frac{D_1}{D_1 + D_2}\Big[h_w\, D_2 - \{f_{w_} + \frac{D_2}{D_1}\,(f_w - f_{w_})\}\Big] \qquad (11) \\
&= \frac{D_1}{D_1 + D_2}\Big[h_w\, D_2 - \{f_w + (1 - \frac{D_2}{D_1})\,(f_{w_} - f_w)\}\Big]. \qquad (12)
\end{aligned}$$

If $f_{w\_} \leq f_w$, then $\frac{D_2}{D_1}\,(f_w - f_{w\_}) \geq 0$. Using inequality (11), the cost excess is bounded by $\frac{D_1}{D_1+D_2}\Big[h_w\, D_2 - f_{w\_}\Big]$ and (6) follows. Alternatively, if $f_{w\_} > f_w$, since $D_2 \leq D_1$, $(1 - \frac{D_2}{D_1})\,(f_{w\_} - f_w) \geq 0$. Using inequality (12), (6) follows.

5. Supplier w has only capacity in period 1, while $w\_$ has enough capacity in both periods and produces both demands in period 1.

   The cost excess bound (6) will be derived using the analysis in Case 4. First, observe that once the demands are allocated to both suppliers, the value of $C_{\mathbf{S}}^{\mathrm{M}}$ will be the same as in Case 4, since $w\_$ has got enough capacity in period 2 to produce D_2. Second, notice that in the proof of Case 4, $C_{w\_}$ is always used as an upper bound of C_w. Therefore, (6) follows here too by noticing that $C_{w\_}$ can be bounded from above by $2\,f_{w\_} + p_{w\_}(D_1{+}D_2)$, the value of $C_{w\_}$ in Case 4.

6. Supplier w has capacity in both periods but the capacity in period 1 is not enough to produce both demands, while $w\_$ has only capacity in period 1.

   It is easy to see that this case is the mirror one to Case 4. We will again distinguish two cases depending on the supplier in charge of demand D_2 in $C^{\mathrm{M}}_{\mathbf{S}}$. For ease of presentation, we will exchange the order of the assignments, analyzing first the case in which D_2 is allocated to $w\_$ in $C^{\mathrm{M}}_{\mathbf{S}}$.

   (a) If D_2 is allocated to $w\_$ in $C^{\mathrm{M}}_{\mathbf{S}}$, D_2 will be produced in period 1. The cost excess can be bounded using (7) where $\Pi_w = f_w$ and $\Pi_{w\_} = h_{w\_}\,D_2$. Thus,

$$\begin{aligned} C_w - C^{\mathrm{M}}_{\mathbf{S}} &= (p_w - p_{w\_})\,D_2 + f_w - f_{w\_} - h_{w\_}D_2 \\ &\leq \frac{D_2}{D_1{+}D_2}\,(f_{w\_} - 2\,f_w + h_{w\_}\,D_2) + f_w - f_{w\_} - h_{w\_}D_2 \\ &= (1 - \frac{2\,D_2}{D_1{+}D_2})\,f_w + (\frac{D_2}{D_1{+}D_2} - 1)\,(f_{w\_} + h_{w\_}D_2) \\ &\leq \frac{D_1 - D_2}{D_1{+}D_2}\,f_w \\ &\leq f_w \end{aligned}$$

   and (6) follows.

   (b) If D_2 is assigned to w in $C^{\mathrm{M}}_{\mathbf{S}}$, D_2 will be produced in period 2.

   If $p_{w\_} \geq p_w$, the cost excess is bounded by

$$C_w - C^{\mathrm{M}}_{\mathbf{S}} \;\leq\; f_w - f_{w\_}.$$

   Now, suppose that $p_w > p_{w\_}$.

   i. If $D_2 > D_1$, we have that

$$\begin{aligned} C_{w\_} &\geq C_w \\ f_{w\_} + p_{w\_}\,(D_1{+}D_2) + h_{w\_}\,D_2 &\geq 2\,f_w + p_w\,(D_1{+}D_2) \\ f_{w\_} + h_{w\_}\,D_2 &\geq 2\,f_w + (p_w - p_{w\_})\,(D_1{+}D_2) \quad \text{Since } D_2 > D_1 : \\ f_{w\_} + h_{w\_}\,D_2 &\geq 2\,(f_w + (p_w - p_{w\_})\,D_1) \\ -f_w + \frac{1}{2}\,(f_{w\_} + h_{w\_}\,D_2) &\geq (p_w - p_{w\_})\,D_1. \end{aligned} \tag{13}$$

The cost excess now can be bounded using (13)

$$
\begin{aligned}
C_w - C^{\mathrm{M}}_{\mathbf{S}} &= (f_w + p_w\, D_1) - (f_{w\_} + p_{w\_} D_1)\\
&= (f_w - f_{w\_}) + (p_w - p_{w\_})D_1\\
&\leq (f_w - f_{w\_}) - f_w + \frac{1}{2}\,(f_{w\_} + h_{w\_}\, D_2)\\
&= -f_{w\_} + \frac{1}{2}\,(f_{w\_} + h_{w\_}\, D_2)\\
&= \frac{1}{2}\,(h_{w\_}\, D_2 - f_{w\_}),
\end{aligned}
$$

and (6) follows.

ii. Otherwise, $D_2 \leq D_1$.
Inequality (7) will be used to bound the cost excess

$$
\begin{aligned}
C_w - C^{\mathrm{M}}_{\mathbf{S}} &= (p_w - p_{w\_})D_1 + (f_w - f_{w\_})\\
&\leq \frac{D_1}{D_1+D_2}\,(f_{w\_} - 2\,f_w + h_{w\_}\, D_2) + (f_w - f_{w\_})\\
&= \frac{D_1}{D_1+D_2}\left[h_{w\_}\, D_2 + f_{w\_} - 2\,f_w + \frac{D_1+D_2}{D_1}\,(f_w - f_{w\_})\right]\\
&= \frac{D_1}{D_1+D_2}\left[h_{w\_}\, D_2 + f_{w\_} - 2\,f_w + (1 + \frac{D_2}{D_1})\,(f_w - f_{w\_})\right]\\
&= \frac{D_1}{D_1+D_2}\left[h_{w\_}\, D_2 - f_w + \frac{D_2}{D_1}\,(f_w - f_{w\_})\right] \qquad (14)\\
&= \frac{D_1}{D_1+D_2}\left[h_{w\_}\, D_2 - f_{w\_} + (\frac{D_2}{D_1} - 1)\,(f_w - f_{w\_})\right]. \qquad (15)
\end{aligned}
$$

If $f_w \leq f_{w\_}$, then $\frac{D_2}{D_1}\,(f_w - f_{w\_}) \leq 0$. Using inequality (14), (6) follows. Alternatively, if $f_{w\_} < f_w$, using that $D_2 \leq D_1$, $(\frac{D_2}{D_1} - 1)\,(f_w - f_{w\_}) \leq 0$. Using inequality (15), (6) follows.

7. Supplier w has capacity in both periods but the capacity in period 1 is not enough to produce both demands, while $w\_$ has enough capacity in both periods and produces both demands in period 1.

   In C_w and $C^{\mathrm{M}}_{\mathbf{S}}$, D_2 will be produced in period 2 and therefore no inventory holding costs are incurred as in Case 2. With respect to $C_{w\_}$, there is an alternative solution, at least as expensive, in which each demand is produced in its own period as in Case 2. Now it is easy to see that the cost excess bound derived in Case 2 applies here too.

8. Supplier w has enough capacity in both periods and produces both demands in period 1, while $w\_$ has only capacity in period 1.

   The proof of this case flows as in Case 6.

9. Supplier w has enough capacity in both periods and produces both demands in period 1, while $w\_$ has capacity in both periods but the capacity in period 1 is not enough to produce both demands.

   The argument here is similar as in Case 7. $\square$

Proof of Proposition 4.5

Consider the following class of problem instances, where the buyer requires demands (D, D). The number of suppliers is equal to four, i.e. $S = 4$, where each supplier faces a unit production cost p and a unit holding cost h. The supplier setup costs and the capacities are given by:

$$f_i = \begin{cases} 0 & \text{for} \quad i = 1, 2 \\ f & \text{for} \quad i = 3, 4 \end{cases}$$
$$b_{it} = \begin{cases} D & \text{for} \quad i = 1, 2, \quad t = i \\ 2\,D & \text{for} \quad i = 3, 4, \quad t = 1 \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where parameters f, p, and h are all positive.

Let us first look at the case of the Standard auction. Suppliers 3 and 4 have identical cost and capacity structures, and are the only suppliers who have sufficient capacity to individually produce the demands. Therefore, in the Standard auction the winner is either supplier 3 or 4. Without loss of generality, we assume that it is supplier 3 with:

$$C_{\mathbf{S}} = C_3 = f + p\,(D + D) + hD = f + 2\,p\,D + hD \quad [w = 3].$$

In the Multiple Winner auction, it is easy to see that the optimal allocation consists of assigning D_t to supplier t in period t, for $t = 1, 2$. The cost of this allocation is:

$$C_{\mathbf{S}}^{\mathrm{M}} = (0 + pD) + (0 + pD) = 2\,p\,D \quad [w_1 = 1,\ w_2 = 2].$$

In the Standard auction, in the absence of supplier 3, supplier 4 delivers the demands with the same costs, i.e., $J = C_{\mathbf{S}\setminus\{3\}} = C_4 = f + 2\,p\,D + hD$. The cost excess is equal to

$$
\begin{aligned}
J^{\mathrm{M}} - J &= C^{\mathrm{M}}_{\mathbf{S}\setminus\{1\}} + C^{\mathrm{M}}_{\mathbf{S}\setminus\{2\}} - C^{\mathrm{M}}_{\mathbf{S}} - C_{\mathbf{S}\setminus\{3\}} \\
&= C^{\mathrm{M}}_{\mathbf{S}\setminus\{1\}} + C^{\mathrm{M}}_{\mathbf{S}\setminus\{2\}} - C^{\mathrm{M}}_{\mathbf{S}} - C_4 \\
&= C^{\mathrm{M}}_{\mathbf{S}} + (C^{\mathrm{M}}_{\mathbf{S}\setminus\{1\}} - C^{\mathrm{M}}_{\mathbf{S}}) + (C^{\mathrm{M}}_{\mathbf{S}\setminus\{2\}} - C^{\mathrm{M}}_{\mathbf{S}}) - C_4
\end{aligned}
$$

in the absence of supplier 1, the best option is to reassign D_1 to supplier 3 at an extra cost of f

$$
= C^{\mathrm{M}}_{\mathbf{S}} + f + (C^{\mathrm{M}}_{\mathbf{S}\setminus\{2\}} - C^{\mathrm{M}}_{\mathbf{S}}) - C_4
$$

in the absence of supplier 2, we must reassign D_2 to supplier 3 at an extra cost of $f + h\,D$

$$
\begin{aligned}
&= C^{\mathrm{M}}_{\mathbf{S}} + f + f + h\,D - C_4 \\
&= (2\,p\,D) + (2\,f + h\,D) - (f + 2\,p\,D + h\,D) \\
&= f,
\end{aligned}
$$

and the desired result follows. $\square$

Proof of Proposition 4.6

Consider the following class of problem instances, where the buyer requires demands (D_1, D_2), where $D_1 \geq D_2$, and thus

$$
\max\left\{\frac{D_1}{D_1+D_2}, \frac{1}{2}\right\} = \begin{cases} \frac{D_1}{D_1+D_2} & \text{if } D_1 > D_2 \\ \frac{1}{2} & \text{if } D_1 = D_2. \end{cases}
$$

The number of suppliers is equal to two, i.e., $S = 2$. Both suppliers face setup costs equal to zero and unit holding costs equal to h. The supplier unit production costs and capacities are given by:

$$
p_i = \begin{cases} p & \text{for} \quad i = 1 \\ p + h\,\frac{D_2}{D_1+D_2} & \text{for} \quad i = 2 \end{cases}
$$

$$
b_{it} = \begin{cases} D_1 + D_2 & \text{for} \quad i = 1, \quad t = 1 \\ D_t & \text{for} \quad i = 2, \quad t = 1, 2 \\ 0 & \text{otherwise,} \end{cases}
$$

where parameters p and h are both positive.

Let us first look at the Standard auction. Because of the capacity structure supplier 1 has to produce both demands in period 1, incurring a total cost equal to

$$C_1 = 0 + (p_1)\, D_1 + (p_1 + h_1)\, D_2 = p\,(D_1 + D_2) + h\, D_2,$$

while supplier 2 must produce each demand in its own period, incurring a total cost equal to

$$C_2 = (0 + p_2\, D_1) + (0 + p_2\, D_2) = (p + h\, \frac{D_2}{D_1 + D_2})\,(D_1 + D_2).$$

It is easy to see that $C_1 = C_2$ and, without loss of generality, we assume that the winner of this auction is supplier 1, i.e.:

$$C_{\mathbf{S}} = C_1 \quad [w = 1].$$

In the Multiple Winner auction, and because the setup costs are equal to zero, each demand will be allocated to the supplier with the lowest total unit cost. Therefore:

$$\begin{aligned} C_{\mathbf{S}}^{\mathrm{M}} &= \min\{p,\ p + h\, \frac{D_2}{D_1 + D_2}\}\, D_1 + \min\{p + h,\ p + h\, \frac{D_2}{D_1 + D_2}\}\, D_2 \\ &= p\, D_1 + (p + h\, \frac{D_2}{D_1 + D_2})\, D_2 \\ &= p\,(D_1 + D_2) + h\, \frac{D_2}{D_1 + D_2}\, D_2. \end{aligned}$$

Since there are only two suppliers and the winner of the Standard auction is supplier 1, the cost excess is given by $C_1 - C_{\mathbf{S}}^{\mathrm{M}}$:

$$\begin{aligned} J^{\mathrm{M}} - J &= C_1 - C_{\mathbf{S}}^{\mathrm{M}} \\ &= (p\,(D_1 + D_2) + h\, D_2) - (p\,(D_1 + D_2) + h\, \frac{D_2}{D_1 + D_2}\, D_2) \\ &= h\,(1 - \frac{D_2}{D_1 + D_2})\, D_2 \\ &= h\, \frac{D_1}{D_1 + D_2}\, D_2. \end{aligned}$$

Now the desired result follows when $h = H$. $\square$

Proof of Proposition 4.8

We need to show that if the setup costs are supplier-independent the new bound for the cost excess is equal to

$$\max\{f,\, \frac{D_1}{D_1 + D_2}\Big[H\, D_2 - f\Big]\}. \tag{17}$$

As in the proof of Theorem 4.1, it remains to show that the bound is valid when $\mathbf{W} = \{w, w\_\}$. The analysis of this case was approached in the proof of Theorem 4.1 by distinguishing nine cases depending on the capacity faced by w and $w\_$ and the timing of the production of both demands in C_w and $C_{w\_}$. In the following, we focus on the cases in which the bound obtained in the proof of Theorem 4.1 was equal to $\frac{1}{2}\Big[H\,D_2 - f\Big] > \frac{D_1}{D_1+D_2}\Big[H\,D_2 - f\Big]$, i.e., Case 4(b), Case 6(b) and Case 8(b).

We will analyze one of these cases and the rest can be handled in a similar way. Consider Case 4(b) with $p_{w\_} \leq p_w$, then (17) follows. Now consider Case 4(b) with $p_{w\_} > p_w$. There, we distinguished two subcases depending on whether $D_2 > D_1$. If the setup costs are supplier-independent, the bound derived for subcase $D_2 \leq D_1$ is valid for the $D_2 > D_1$ subcase too. Indeed, we know that

$$\begin{aligned} C_w - C^{\mathrm{M}}_{\mathbf{S}} &= (p_w - p_{w\_})\,D_2 + h_w\,D_2 - f_{w\_} \\ &\leq \frac{D_2}{D_1+D_2}\,\{2\,f_{w\_} - f_w - h_w\,D_2\} + h_w\,D_2 - f_{w\_} \\ &= \frac{D_2}{D_1+D_2}\,\{f - h_w\,D_2\}\,D_2 + h_w\,D_2 - f \\ &= \frac{D_1}{D_1+D_2}\,\{h_w\,D_2 - f\} \\ &\leq \frac{D_1}{D_1+D_2}\,\{H\,D_2 - f\}, \end{aligned}$$

and (17) follows.

Finally, the tightness of the bound (17) is a straightforward corollary from Propositions 4.5 and 4.6. We simply need to notice that, in the class of problem instances used in both propositions, the setup costs are stationary. $\square$

Proof of Proposition 4.11

This proposition can be proved in a similar fashion to Proposition 4.8. We need to show that if capacity is only available in one period the new bound for the cost excess is equal to

$$\max\{\max_{i\,\in\,\mathbf{S}} f_i, \frac{D_1}{D_1+D_2}\Big[H\,D_2 - \min_{i\,\in\,\mathbf{S}} f_i\Big]\}. \tag{18}$$

Again, it remains to show that the bound is valid when $\mathbf{W} = \{w, w\_\}$. Since capacity is only available in one period, w and $w\_$ will enjoy their capacity in period 1. This corresponds to Case 1 in the proof of Theorem 4.1. From that analysis, the result follows trivially.

To derive the tightness of the bound (18), we will use the class of problem instances introduced in Proposition 4.6, but where supplier 2 has the same capacity pattern as supplier 1. This means that this class of problem instances satisfies the *capacity in only one period* condition. Additionally, we impose that the unit inventory holding cost of supplier 2 is equal to zero.

Let us first look at the Standard auction. Now both suppliers have to produce both demands in period 1, incurring a total cost equal to

$$\begin{aligned} C_1 &= 0 + (p_1)\, D_1 + (p_1{+}h_1)\, D_2 = p\,(D_1{+}D_2) + h\, D_2 \\ C_2 &= 0 + (p_2)\, D_1 + (p_2{+}h_2)\, D_2 = (p + h\,\frac{D_2}{D_1{+}D_2})\,(D_1{+}D_2) = p\,(D_1{+}D_2) + h\, D_2. \end{aligned}$$

Again, $C_1 = C_2$ and, without loss of generality, the winner of this auction is supplier 1.

In the Multiple Winner auction, and because the setup costs are equal to zero, each demand will be allocated to the supplier with the lowest total unit cost. Therefore:

$$\begin{aligned} C^{\mathrm{M}}_{\mathbf{S}} &= \min\{p, p + h\,\frac{D_2}{D_1{+}D_2}\}\, D_1 + \min\{p+h, p + h\,\frac{D_2}{D_1{+}D_2}\}\, D_2 \\ &= p\, D_1 + (p + h\,\frac{D_2}{D_1{+}D_2})\, D_2. \end{aligned}$$

The rest of the proof flows as in Proposition 4.8, and the desired result follows. $\square$

Proof of Proposition 5.1

From Lemma A.4, it remains to show that if w and $w\_$ are both winners of the Multiple Winner auction, there is an alternative allocation where both period demands are assigned to the same supplier. Suppose that w and $w\_ \in \mathbf{W}$. Let $s \in \arg\min\{p_w, p_{w\_}\}$. In the remaining of the proof, we will show that there exists an alternative allocation in the Multiple Winner auction such that the period demands in $\mathbf{T}_{s'}$ are assigned to supplier s, where $s' \in \{w, w\_\}\backslash\{s\}$. Thus, in this alternative allocation either w or $w\_ \notin \mathbf{W}$, and the desired result follows.

Using the monotonicity condition, we have that $s \in \arg\min\{p_w + (t{-}1)\, h_w, p_{w\_} + (t{-}1)\, h_{w\_}\}$, for all $t = 2, \ldots, T$. This means that, in terms of total unit costs associated with each period demand, supplier s is at least as attractive as s'. Together with the fact that capacity is only available in period 1 for both s and s', this means that $C_{s'}(\mathbf{T}_{s'}) - f_{s'} \geq C_s(\mathbf{T}_{s'}) - f_s$. Now we have

$$C^{\mathrm{M}}_{\mathbf{S}} = \sum_{i \in \mathbf{W}} C^{\mathrm{M}}_{\mathbf{S},i}$$

$$
\begin{aligned}
&= \sum_{i\in\mathbf{W},i\neq s,s'} C^{\mathrm{M}}_{\mathbf{S},i} + C^{\mathrm{M}}_{\mathbf{S},s} + C^{\mathrm{M}}_{\mathbf{S},s'} \quad \text{Since capacity is only available in period 1:}\\
&= \sum_{i\in\mathbf{W},i\neq s,s'} C^{\mathrm{M}}_{\mathbf{S},i} + C_s(\mathbf{T}_s) + C_{s'}(\mathbf{T}_{s'})\\
&= \sum_{i\in\mathbf{W},i\neq s,s'} C^{\mathrm{M}}_{\mathbf{S},i} + f_s + (C_s(\mathbf{T}_s) - f_s) + f_{s'} + (C_{s'}(\mathbf{T}_{s'}) - f_{s'})\\
&\geq \sum_{i\in\mathbf{W},i\neq s,s'} C^{\mathrm{M}}_{\mathbf{S},i} + f_s + (C_s(\mathbf{T}_s) - f_s) + f_{s'} + (C_s(\mathbf{T}_{s'}) - f_s)\\
&\geq \sum_{i\in\mathbf{W},i\neq s,s'} C^{\mathrm{M}}_{\mathbf{S},i} + C_s(\mathbf{T}_s) + C_s(\mathbf{T}_{s'}) - f_s\\
&= \sum_{i\in\mathbf{W},i\neq s,s'} C^{\mathrm{M}}_{\mathbf{S},i} + C_s(\mathbf{T}_s \cup \mathbf{T}_{s'}),
\end{aligned}
$$

and thus $C^{\mathrm{M}}_{\mathbf{S}} = \sum_{i\in\mathbf{W},i\neq s,s'} C^{\mathrm{M}}_{\mathbf{S},i} + C_s(\mathbf{T}_s \cup \mathbf{T}_{s'})$, and thus we have found an alternative allocation in which s' is not a winner of the Multiple Winner auction. □

Proof of Proposition 5.4

Consider the following class of problem instances, where the buyer requires demand vector $\mathbf{D} \in \mathbb{R}^T$, such that $D_1 > D_2 > \ldots > D_{T-1} > D_T > 1$. The number of suppliers exceeds the number of periods by two, i.e., $S = T+2$, where each supplier faces a setup cost f and a unit holding cost h in each period. The supplier unit production costs and capacities are given by:

$$
\begin{aligned}
p_i &= \begin{cases} p + (T-i)\,f & \text{for} \quad i \leq T \\ p + Tf & \text{for} \quad i = T+1, T+2 \end{cases}\\
b_{it} &= \begin{cases} D_t & \text{for} \quad i \leq T, \quad t = i \\ \sum_{\tau=1}^{T} D_\tau & \text{for} \quad i = T+1, T+2, \quad t = 1 \\ 0 & \text{otherwise,} \end{cases}
\end{aligned}
\tag{19}
$$

where parameters f, p, and h are all positive.

Now we will determine the structure of the optimal allocation in the Standard auction and in the Multiple Winner auction. First have a look at the case of the Standard auction. Suppliers $T+1$ and $T+2$ have identical cost and capacity structures, and are the only suppliers who have sufficient capacity to individually produce the demand vector. Therefore, in the Standard auction the winner is either supplier $T+1$ or $T+2$. Without loss of generality, we assume that it is supplier $T+1$ with:

$$C_{\mathbf{S}} \;=\; f + \sum_{t=1}^{T}[p + Tf + (t-1)\,h]\,D_t \quad [w = T+1]. \tag{20}$$

In the Multiple Winner auction, it can be shown by contradiction that the optimal allocation consists of assigning D_t to supplier t in period t, for each $t \in \mathbf{T}$ (see the Appendices) with

$$\begin{aligned} C^{\mathrm{M}}_{\mathbf{S}} &= \sum_{t=1}^{T} (f + [p + (T-t)\,f]\,D_t), \\ &= Tf + \sum_{t=1}^{T}[p + (T-t)\,f]\,D_t, \end{aligned} \tag{21}$$

and $\mathbf{W} = \mathbf{T} = \{1, \ldots, T\}$.

Now we will determine the structure of the optimal allocation in the Multiple Winner auction after the elimination of supplier $t \in \mathbf{W}$. It can be shown by contradiction that each demand D_τ, with $\tau \neq t$, will be assigned to supplier τ in period τ, while the demand D_t will be allocated without loss of generality to supplier $T+1$ in period 1 (see the Appendices). This means that in determining $C^{\mathrm{M}}_{\mathbf{S}\setminus\{t\}}$ all demands, except for D_t, are allocated in the same manner as in determining $C^{\mathrm{M}}_{\mathbf{S}}$. Therefore,

$$\begin{aligned} C^{\mathrm{M}}_{\mathbf{S}\setminus\{t\}} - C^{\mathrm{M}}_{\mathbf{S}} &= (f + [p + Tf]\,D_t + (t-1)\,h\,D_t) - (f + [p + (T-t)f]\,D_t) \\ &= [tf + (t-1)\,h]\,D_t \qquad \text{for each } t \in \mathbf{W}. \end{aligned} \tag{22}$$

In the Standard auction, in the absence of supplier $T+1$, supplier $T+2$ delivers the demands with the same costs, i.e., $J = C_{\mathbf{S}\setminus\{T+1\}} = C_{T+2} = f + \sum_{t=1}^{T}[p + Tf + (t-1)\,h]\,D_t$. The cost excess is equal to

$$\begin{aligned} J^{\mathrm{M}} - J &= C^{\mathrm{M}}_{\mathbf{S}} + \sum_{i\in\mathbf{W}}(C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - C^{\mathrm{M}}_{\mathbf{S}}) - C_{\mathbf{S}\setminus\{T+1\}} \\ &= C^{\mathrm{M}}_{\mathbf{S}} + \sum_{i\in\mathbf{W}}(C^{\mathrm{M}}_{\mathbf{S}\setminus\{i\}} - C^{\mathrm{M}}_{\mathbf{S}}) - C_{T+2} \\ &= C^{\mathrm{M}}_{\mathbf{S}} + \sum_{t=1}^{T}(C^{\mathrm{M}}_{\mathbf{S}\setminus\{t\}} - C^{\mathrm{M}}_{\mathbf{S}}) - C_{T+2}, \quad \text{and from (21) and (22) :} \\ &= (Tf + \sum_{t=1}^{T}[p + (T-t)\,f]\,D_t) + \sum_{t=1}^{T}[tf + (t-1)\,h]\,D_t - C_{T+2} \end{aligned}$$

$$
\begin{aligned}
&= Tf + \sum_{t=1}^{T} [p + Tf + (t-1)\,h]\, D_t - C_{T+2} \\
&= (T-1)\, f \\
&= (\min\{S,T\} - 1)\, f,
\end{aligned}
$$

and the desired result follows. $\square$

Proof of Proposition 5.4: The structure of $C^{\mathrm{M}}_{\mathbf{S}}$ and $C^{\mathrm{M}}_{\mathbf{S}\setminus\{t\}}$

Here we will discuss in detail the structure of the optimal allocation in the Multiple Winner auction and in the Multiple Winner auction after the elimination of supplier $t \in \mathbf{W}$.

Before we analyze these allocations, recall that suppliers $T{+}1$ and $T{+}2$ have identical cost and capacity structures. When required, without loss of generality we will choose supplier $T+1$. Also notice that, for this class of problem instances, the setup and the unit inventory holding costs are the same for all suppliers; therefore when comparing two (supplier, period) combinations we only need to discuss the unit production costs. We have that: (a) the only supplier that can produce demand D_t in period t is supplier t, for all $t \in \{2, \dots, T\}$, and (b) supplier 1 is the one having the cheapest unit production costs in period 1. From observations (a) and (b), we can derive that (c) supplier t is the cheapest option when producing D_t in period t, for all $t \in \{1, \dots, T\}$.

We will start with $C^{\mathrm{M}}_{\mathbf{S}}$. In this paragraph, we will show that in the optimal allocation for the Multiple Winner auction, every demand will be produced in the period in which it is demanded. From this and observation (c) the desired result follows, i.e., the optimal allocation in the Multiple Winner auction consists of assigning D_t to supplier t in period t, for each $t \in \{1, \dots, T\}$. Now suppose that, in the optimal allocation of the Multiple Winner auction, there exists at least one demand that is produced in advance. Let $\hat{t}$ be the largest element in $\{1, \dots, T\}$ such that $D_{\hat{t}}$ is produced in advance. This means that any demand D_t, with $t > \hat{t}$, will be produced in the period in which it is demanded. Since $D_{\hat{t}}$ is produced in advance and the demands in the future are produced in the respective periods in which they are demanded, we know that supplier $\hat{t}$ does not produce in period $\hat{t}$. In the following, we will show that by allocating demand $D_{\hat{t}}$ to supplier $\hat{t}$ in period $\hat{t}$, we obtain a feasible allocation which is cheaper than the current one, and this will yield a contradiction. First, it is easy to see that this is a feasible allocation, since supplier $\hat{t}$ does not produce during period $\hat{t}$. Second, we will show that this yields a cheaper allocation. Because demand $D_{\hat{t}}$ is produced some time before period $\hat{t}$, the unit production cost paid for this demand will be at least $p + (T - (\hat{t} - 1))f$. Therefore the variable production costs incurred will be at least $[p + (T - (\hat{t} - 1))\, f]\, D_{\hat{t}}$. Since $D_{\hat{t}} > 1$, we have that this cost is greater than $f + [p + (T - \hat{t})\, f]\, D_{\hat{t}}$, i.e., the total cost of producing $D_{\hat{t}}$ by supplier $\hat{t}$ in period $\hat{t}$ (including setup costs), a contradiction.

We will now discuss $C^{\mathrm{M}}_{\mathbf{S}\setminus\{t\}}$, $t \in \{1, \ldots, T\}$. Since suppliers $T{+}1$ and $T{+}2$ are identical, we will discard supplier $T+2$ from the rest of the proof. Using a similar argument as for $C^{\mathrm{M}}_{\mathbf{S}}$, we have that demand D_τ will be produced in time by supplier τ, for all $\tau > t$. In the following, we will show that demand D_t will be produced in period 1 by supplier $T+1$. Using a similar argument as for $\tau > t$, this will imply that, for all $\tau < t$, demand D_τ will also be produced in time by supplier τ, and the desired result will follow. Now consider the allocation of demand D_t. Suppose that D_t is not assigned to supplier $T+1$, then it will be produced by supplier $\tilde{t}$ in period $\tilde{t}$ with $\tilde{t} \in \{1, \ldots, t-1\}$, while, and using a similar argument as in the case of $C^{\mathrm{M}}_{\mathbf{S}}$, the demands indexed by $\{\tilde{t}+1, \ldots, t-1\}$ will be produced in the period in which they are demanded. Because of capacity constraints of supplier $\tilde{t}$, demand $D_{\tilde{t}}$ will need to be produced in advance. Eventually, there will be a single demand in the future that will need to be produced in period 1, by supplier 1 because of observation (b). Again, due to capacity constraints, D_1 will need to be assigned to supplier $T+1$. However, the unit production cost of supplier 1 is cheaper than that of supplier $T+1$. Therefore, by exchanging the assignments of suppliers 1 and $T+1$, we obtain a feasible allocation that is cheaper than the current one, a contradiction. $\square$

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