

Appendix

to accompany article in
Management Science

“When do employees become entrepreneurs?”

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Table A2: Key notation

A	Employee's choice to focus on core task
a	Returns to core task
B	Employee's choice to explore new idea
β_x, β_y	Fraction of utility that constitutes a private benefit
δ^x	Policy of developing innovations externally
δ^y	Policy of developing innovations internally
δ^0	Policy of not developing innovations (shelving)
EIP	Regime where intellectual property rights belong to employee
FIP	Regime where intellectual property rights belong to firm
ϕ	Probability that returns a occur when core prospects are weak
γ	Probability that core prospects are strong at date 1
$\hat{\gamma}_1, \hat{\gamma}_2$	Critical values of γ , satisfying $1 > \hat{\gamma}_1 > \hat{\gamma}_2 \geq 0$
λ	Probability that employee obtains an idea at date 1
$\bar{\lambda}$	Critical value of λ
p	Probability that an idea turns into feasible innovation
σ^-	Equilibrium where employee never explores
σ^*	Equilibrium where employee explores only if core prospects are weak
σ^+	Equilibrium where employee explores excessively (even if core prospects are strong)
w_A, w_y, w_x	Compensation for generating outcomes a, y or x
x	Utility to an external venture (start-up or spin-off)
$\hat{x}_1, \hat{x}_2, \hat{x}_3$	Critical values of x , satisfying $0 < \hat{x}_1 < \hat{x}_2 < y < \hat{x}_3$
y	Utility to an internal venture
z	Utility of employee in an internal venture (given by $Max[\beta_y y, x]$)
Remember that a bar over a probability means its complement, e.g., $\bar{\lambda} = 1 - \lambda$	

Proof of Proposition 2

It is useful to restate the firm's utilities from equations (2) through (7) in the main text.

$$\begin{aligned}
 U(\sigma^-, \delta^y) &= (\gamma + \bar{\gamma}\phi)(a - pz\phi^{-1}) \\
 U(\sigma^*, \delta^y) &= \lambda\bar{\gamma}(py - pz) + (\gamma + \bar{\lambda}\bar{\gamma}\phi)(a - pz) \\
 U(\sigma^+, \delta^y) &= \lambda(py - pz) + \bar{\lambda}(\gamma + \bar{\gamma}\phi)a \\
 U(\sigma^-, \delta^x) &= (\gamma + \bar{\gamma}\phi)(a - px\phi^{-1}) \\
 U(\sigma^*, \delta^x) &= (\gamma + \bar{\lambda}\bar{\gamma}\phi)(a - px) \\
 U(\sigma^+, \delta^x) &= \bar{\lambda}(\gamma + \bar{\gamma}\phi)a
 \end{aligned}$$

We immediately note that $U(\sigma^+, \delta^y) > U(\sigma^+, \delta^x)$, i.e., (σ^+, δ^x) is dominated. Using $\lambda < \hat{\lambda}$, we only need to compare three policies: (σ^-, δ^x) , (σ^*, δ^x) and (σ^+, δ^y) . Consider first the case of $\beta_y y \leq x \leq y$, then $pz = px$. To see that (σ^-, δ^x) is dominated by (σ^+, δ^y) , note that $U(\sigma^-, \delta^y) = U(\sigma^-, \delta^x)$ and that the proof of Proposition 1 already establishes that $U(\sigma^+, \delta^y) > U(\sigma^-, \delta^y)$. Moreover, (σ^*, δ^x) is also dominated by (σ^+, δ^y) , since for $pz = px$ we have $U(\sigma^*, \delta^y) > U(\sigma^*, \delta^x)$, and the proof of Proposition 1 establishes that $U(\sigma^+, \delta^y) > U(\sigma^*, \delta^y)$. It follows that (σ^+, δ^y) is optimal for $\beta_y y \leq x \leq y$. For the remainder of the proof we focus on $x < \beta_y y$.

Consider the effect of x on the firm's utility from the three candidate policies.

We note that $\frac{dU(\sigma^-, \delta^x)}{dx} = -(\gamma + \bar{\gamma}\phi)\frac{p}{\phi} < \frac{dU(\sigma^*, \delta^x)}{dx} = -p(\gamma + \bar{\lambda}\bar{\gamma}\phi) < \frac{dU(\sigma^+, \delta^y)}{dx} = 0$ (using $x < \beta_y y$). This implies that there exists a critical value \hat{x}_- , such that $U(\sigma^*, \delta^x) > U(\sigma^-, \delta^x) \Leftrightarrow x > \hat{x}_-$. Similarly, there exists \hat{x}_+^+ such that $U(\sigma^+, \delta^y) > U(\sigma^-, \delta^x) \Leftrightarrow x > \hat{x}_+^+$ and \hat{x}_*^+ such that $U(\sigma^+, \delta^y) > U(\sigma^*, \delta^x) \Leftrightarrow x > \hat{x}_*^+$. Straightforward calculations reveal that the critical values are given by $\hat{x}_1 \equiv \hat{x}_-^* = \frac{\lambda}{p} \frac{\bar{\gamma}\phi a}{\gamma\phi\phi^{-1} + \lambda\bar{\gamma}}$, $\hat{x}_-^+ = \frac{\lambda}{p} \frac{(\gamma + \bar{\gamma}\phi)a - (py - pz)}{\gamma\phi^{-1} - \bar{\gamma}}$ and $\hat{x}_2 \equiv \hat{x}_*^+ = \frac{\lambda}{p} \frac{\gamma a - (py - pz)}{\gamma + \bar{\lambda}\bar{\gamma}}$. It is easy to verify that $\frac{d\hat{x}_1}{da} > 0$, $\frac{d\hat{x}_1}{dp} < 0$, $\frac{d\hat{x}_1}{d\phi} > 0$, $\frac{d\hat{x}_2}{da} > 0$, $\frac{d\hat{x}_2}{dp} < 0$, $\frac{d\hat{x}_2}{d\phi} = 0$.

Consider $\Delta_1 \equiv U(\sigma^*, \delta^x) - U(\sigma^+, \delta^y)$ evaluated at \hat{x}_-^* . If $\Delta_1 > 0$, then $U(\sigma^-, \delta^x) = U(\sigma^*, \delta^x) > U(\sigma^+, \delta^y)$ at \hat{x}_-^* . Since $\frac{dU(\sigma^-, \delta^x)}{dx} < \frac{dU(\sigma^+, \delta^y)}{dx}$ this implies $\hat{x}_-^+ > \hat{x}_-^*$. Since $\frac{dU(\sigma^*, \delta^x)}{dx} < \frac{dU(\sigma^+, \delta^y)}{dx}$ it also implies $\hat{x}_*^+ > \hat{x}_-^*$. Moreover, $\frac{dU(\sigma^-, \delta^x)}{dx} < \frac{dU(\sigma^*, \delta^x)}{dx}$ implies $\hat{x}_-^+ < \hat{x}_*^+$. Clearly $\hat{x}_-^* > 0$. Moreover, we have already shown that $U(\sigma^+, \delta^y) > U(\sigma^*, \delta^x)$ at $x = \beta_y y$, so that $\hat{x}_*^+ < \beta_y y$. Thus $\Delta_1 > 0$ implies $0 < \hat{x}_-^* < \hat{x}_-^+ < \hat{x}_*^+ < \beta_y y < y$.

We now examine the condition $\Delta_1 > 0$. We have $\Delta_1 = \lambda\gamma a - \lambda py + \lambda pz - (\gamma + \bar{\lambda}\bar{\gamma})p\hat{x}_-^*$. Using $\hat{x}_-^* = \frac{\lambda}{p} \frac{\bar{\gamma}\phi a}{\gamma\bar{\phi}\phi^{-1} + \lambda\bar{\gamma}}$, we obtain after transformations $\Delta_1 > 0 \Leftrightarrow T(\gamma) > 0$ where $T(\gamma) = \gamma^2\bar{\phi}\phi^{-1}a + \lambda\gamma\bar{\gamma}a - \gamma\bar{\gamma}\phi a - \bar{\lambda}\bar{\gamma}\phi a - (\gamma\bar{\phi}\phi^{-1} - \lambda\bar{\gamma})(py - pz)$. For $\gamma \rightarrow 0$ we have $T(\gamma) = -\bar{\lambda}\phi a - \lambda(py - pz) < 0$, and for $\gamma \rightarrow 1$ we have $T(\gamma) = \frac{\bar{\phi}}{\phi}(a - py + pz) > 0$. We define $\hat{\gamma}_1$ so that $T(\hat{\gamma}_1) = 0$. To show that $\hat{\gamma}_1$ is unique, it suffices to show that $\frac{dT(\gamma)}{d\gamma} > 0$ at $T(\hat{\gamma}_1) = 0$. It is tedious but straightforward to verify this condition.

Proof of Proposition 3

Let us first focus on the analysis where the firm can commit. In the main text, we already showed that for (σ^-, δ^0) , the firm sets $w_a = 0$, so that

$$U(\sigma^-, \delta^0) = (\gamma + \bar{\gamma}\phi)a.$$

We also need to re-derive the utility that the firm gets from all other policies. For external developments δ^x , the main difference is that the employee's outside option is no longer px , but $p\beta_x x + w_x$. Since the firm never wants to encourage idea exploration, it is easy to see that $w_x = 0$. Thus, for (σ^-, δ^x) , the firm sets $w_a = p\beta_x x\phi^{-1}$, so that

$$U(\sigma^-, \delta^x) = (\gamma + \bar{\gamma}\phi)(a - p\beta_x x\phi^{-1}).$$

For (σ^*, δ^x) , the firm sets $w_a = p\beta_x x$, so that

$$U(\sigma^*, \delta^x) = (\gamma + \bar{\lambda}\bar{\gamma}\phi)(a - p\beta_x x).$$

For (σ^+, δ^x) , the firm sets $w_a = 0$, so that

$$U(\sigma^+, \delta^x) = \bar{\lambda}(\gamma + \bar{\gamma}\phi)a.$$

We immediately recognize that (σ^-, δ^0) dominates any of the policies with δ^x . What remains to be seen is how (σ^-, δ^0) compares to any strategy with δ^y . From Proposition 1, we only need to consider (σ^+, δ^y) , where the firm sets $w_a = 0$ so that

$$U(\sigma^+, \delta^y) = \lambda(py - p\beta_y y) + \bar{\lambda}(\gamma + \bar{\gamma}\phi)a.$$

Consider $\Delta_2 = U(\sigma^-, \delta^0) - U(\sigma^+, \delta^y)$. After transformations, we have $\Delta_2 = \lambda\gamma a + \lambda\bar{\gamma}\phi a - \lambda(py - p\beta_y y)$. From $\frac{d\Delta_2}{d\gamma} = \lambda\bar{\phi} > 0$, we note that Δ_2 is an increasing function of γ . For $\gamma \rightarrow 1$ we have $\Delta_2 = \lambda(a - py + p\beta_y y) > 0$. We find $\hat{\gamma}_2$ from $\Delta_2 = 0 \Leftrightarrow \hat{\gamma}_2 = \frac{py - \phi a - p\beta_y y}{\bar{\phi}a}$. Note that $\hat{\gamma}_2$ may actually be

negative for $p\beta_y y > py - \phi a$, in which case the condition $\gamma > \hat{\gamma}_2$ is trivially satisfied. To see that $\hat{\gamma}_2 < \hat{\gamma}_1$, we note that Δ_1 can be written as $\Delta_1(\gamma) = \lambda\gamma a - \lambda(py - pz) - (\gamma + \bar{\lambda}\bar{\gamma})p\hat{x}_-^*$. Using $z = p\beta_y y$ and $\Delta_2(\gamma) = \lambda(\phi a - py + p\beta_y y)$ we obtain $\Delta_1(\gamma) = \Delta_2(\gamma) - \lambda\bar{\gamma}\phi a - (\gamma + \bar{\lambda}\bar{\gamma})p\hat{x}_-^*$. At $\hat{\gamma}_2$ we thus have $\Delta_1(\hat{\gamma}_2) = -\lambda\bar{\gamma}\phi a - (\gamma + \bar{\lambda}\bar{\gamma})p\hat{x}_-^* < 0$, and thus $\hat{\gamma}_2 < \hat{\gamma}_1$.

For completeness, we briefly mention two additional policies with shelving, namely (σ^*, δ^0) and (σ^+, δ^0) . These are rather strange policies, since the firm needs to reward the employee for generating innovations that it then shelves. It is easy to see that this is never optimal.

For the case where the firm cannot commit to a development policy, the only policies available for $x < y$ involve δ^y . Hence the analysis is the same as in Proposition 1, except that z is replaced by $\beta_y y$ (since the employee does not have x as an outside option).

Proof of Proposition 4

Consider now the case of $x > y$. We begin with the case where the employee owns the IP. We immediately note that choosing δ^y is a dominated strategy. This is because in order to convince the employee to do an internal venture, the firm has to offer $w_y = x - \beta_y y$, and receives a utility of $y - \beta_y y - w_y = y - x < 0$. The firm is always better off with δ^x , which gives the employee the same utility, but avoids the loss of $y - x$ to the firm. Consider now the δ^x strategy. Using the same reasoning as in Proposition 2, we have

$$\begin{aligned} U(\sigma^-, \delta^x) &= (\gamma + \bar{\gamma}\phi)(a - px\phi^{-1}) \\ U(\sigma^*, \delta^x) &= (\gamma + \bar{\lambda}\bar{\gamma}\phi)(a - px) \\ U(\sigma^+, \delta^x) &= (\bar{\lambda}\gamma + \bar{\lambda}\bar{\gamma}\phi)a. \end{aligned}$$

We note that $U(\sigma^*, \delta^x) > U(\sigma^-, \delta^x) \Leftrightarrow \gamma px\bar{\phi}\phi^{-1} + \bar{\gamma}\bar{\phi}px + \lambda\bar{\gamma}\phi(px - \phi a) > 0$. Moreover, $U(\sigma^+, \delta^x) > U(\sigma^*, \delta^x) \Leftrightarrow \lambda < \frac{(\gamma + \bar{\gamma}\phi)px}{\gamma a + \bar{\gamma}\phi px} \equiv \hat{\lambda}$. It is easy to verify

that $\hat{\lambda} > \bar{\lambda}$. Thus (σ^+, δ^x) is the optimal strategy. Moreover, since δ^x actually requires no commitment, the optimal strategy is the same irrespective of whether the firm is able to commit or not.

Consider now the case where the firm owns the IP. In this case we cannot immediately eliminate the δ^y strategy, since it is possible that the firm prefers δ^y over δ^x , namely when $y - \beta_y y > x - \beta_x x$. To implement σ^- , the firm can use δ^x , δ^y or δ^0 and obtain

$$\begin{aligned} U(\sigma^-, \delta^x) &= (\gamma + \bar{\gamma}\phi)(a - p\beta_x x\phi^{-1}) \\ U(\sigma^-, \delta^y) &= (\gamma + \bar{\gamma}\phi)(a - p\beta_y y\phi^{-1}) \\ U(\sigma^-, \delta^0) &= (\gamma + \bar{\gamma}\phi)a \end{aligned}$$

Clearly, (σ^-, δ^0) is the dominant of those three policies. For the same reason as before, (σ^*, δ^0) also dominates (σ^+, δ^0) and (σ^-, δ^0) . We are thus left with four additional policies. We have

$$\begin{aligned} U(\sigma^*, \delta^x) &= \lambda \bar{\gamma}(px - p\beta_x x) + (\gamma + \bar{\lambda} \bar{\gamma} \phi)(a - p\beta_x x) \\ U(\sigma^*, \delta^y) &= \lambda \bar{\gamma}(py - p\beta_y y) + (\gamma + \bar{\lambda} \bar{\gamma} \phi)(a - p\beta_y y) \\ U(\sigma^+, \delta^x) &= \lambda(px - p\beta_x x) + (\bar{\lambda} \gamma + \bar{\lambda} \bar{\gamma} \phi)a \\ U(\sigma^+, \delta^y) &= \lambda(py - p\beta_y y) + (\bar{\lambda} \gamma + \bar{\lambda} \bar{\gamma} \phi)a \end{aligned}$$

We first note that $U(\sigma^+, \delta^y) > U(\sigma^*, \delta^y) \Leftrightarrow \lambda < \frac{(\gamma + \bar{\gamma} \phi)p\beta_y y}{\gamma(a - py) + (\gamma + \bar{\gamma} \phi)p\beta_y y} = \hat{\lambda}$

and $U(\sigma^+, \delta^x) > U(\sigma^*, \delta^x) \Leftrightarrow \lambda < \frac{(\gamma + \bar{\gamma} \phi)p\beta_x x}{\gamma(a - px) + (\gamma + \bar{\gamma} \phi)p\beta_x x}$. Since

$\frac{(\gamma + \bar{\gamma} \phi)p\beta_x x}{\gamma(a - px) + (\gamma + \bar{\gamma} \phi)p\beta_x x} \geq \hat{\lambda}$, it follows that $U(\sigma^+, \delta^x) > U(\sigma^*, \delta^x)$ for $\lambda < \hat{\lambda}$.

Thus, both (σ^*, δ^y) and (σ^*, δ^x) are dominated by (σ^+, δ^x) . Next, consider the relationship between (σ^+, δ^x) and (σ^+, δ^y) . We have $U(\sigma^+, \delta^x) > U(\sigma^+, \delta^y)$

$\Leftrightarrow x > \frac{1 - \beta_y}{1 - \beta_x} y \equiv \hat{x}_y$. Consider first the case where $x < \hat{x}_y$. To find the

optimal strategy, we only need to compare (σ^-, δ^0) with (σ^+, δ^y) . We note that $U(\sigma^-, \delta^0) > U(\sigma^+, \delta^y) \Leftrightarrow \gamma > \frac{py - \phi a - p\beta_y y}{\bar{\phi} a} = \hat{\gamma}_2$, so that (σ^-, δ^0) is the

optimal strategy for all $x < \hat{x}_y$. For $x > \hat{x}_y$, we compare (σ^-, δ^0) with (σ^+, δ^x) .

We have $U(\sigma^-, \delta^0) > U(\sigma^+, \delta^x) \Leftrightarrow x < \frac{\gamma + \bar{\gamma} \phi a}{1 - \beta_x p} \equiv \hat{x}_3$, so that (σ^-, δ^0) is

optimal for $x < \hat{x}_3$ and (σ^+, δ^x) is optimal for $x < \hat{x}_3$. We immediately note

that $\frac{d\hat{x}_3}{da} > 0$, $\frac{d\hat{x}_3}{dp} < 0$ and $\frac{d\hat{x}_3}{d\phi} > 0$.

We briefly have a closer look at the critical value \hat{x}_3 . We note that at $\gamma = \hat{\gamma}_2$ we have $\hat{x}_3 = \hat{x}_y$, and for all $\gamma > \hat{\gamma}_2$ we have $\hat{x}_3 > \hat{x}_y$. Moreover, note that $\beta_x \geq \beta_y \Leftrightarrow \hat{x}_y \geq y$, so that $\beta_x \geq \beta_y$ is sufficient to ensure that $\hat{x}_3 > y$. One minor difference of the model with $\beta_x < \beta_y$ is that for γ close to $\hat{\gamma}_2$, it is possible that $\hat{x}_3(\hat{\gamma}_2) < y$.

The fact that (σ^+, δ^x) eventually becomes optimal for sufficiently large x is not surprising, since for very large x , idea exploration becomes extremely profitable. The more interesting result is that (σ^+, δ^x) can be optimal for values of x where idea exploration is still inefficient with a strong core. Formally, this is the case as long as $x < \frac{a}{p}$. We note that $\hat{x}_3 < \frac{a}{p} \Leftrightarrow \gamma < 1 - \beta_x \bar{\phi}^{-1}$, so that for any $\gamma \in (\hat{\gamma}_2, 1 - \beta_x \bar{\phi}^{-1})$ there exists a range of values of x (namely $x \in (\hat{x}_3, \frac{a}{p})$),

such that (σ^+, δ^x) is optimal, even though idea exploration is inefficient with a strong core.

As a last step, we need to consider the case where the firm has the IP, but is unable to commit. In this case, any policy with δ^0 is not a credible policy. It immediately follows that (σ^+, δ^y) is optimal for $x < \hat{x}_y$ and (σ^+, δ^x) is optimal

for $x < \hat{x}_y$. Note that for $\beta_x > \beta_y$ we have $\hat{x}_y > y$, and for $\beta_x < \beta_y$ we have $\hat{x}_y > y$

Discussion of model restrictions

The main text uses parameter restrictions on γ , λ and the β 's. We now discuss what happens to the model if we relax these restrictions.

Consider relaxing the assumption $\beta_x = \beta_y$. If the employee owns the IP, β_x never matters for the analysis, since the employee always gets x anyway. If the firm owns the IP, and is able to commit, then the proof of Proposition 4 shows that the critical value \hat{x}_3 remains the same for $\beta_x \neq \beta_y$. Thus, the only change pertains to the case where the firm owns the IP and is unable to commit. In fact, the only difference concerns the critical value at which the firm switches from the intrapreneurial equilibrium (σ^+, δ^y) to the entrepreneurial equilibrium (σ^+, δ^y) . The above proof of Proposition 4 shows that this critical value is given by $x = \frac{1 - \beta_y}{1 - \beta_x} y$. Note that this simplifies to $x = y$ for $\beta_x = \beta_y$. To get an intuition, consider $\beta_x > \beta_y$, so that the employee enjoys a higher private benefit in external ventures. In this case, the firm develops a slight preference for internal ventures, precisely because the employee is less able to extract private benefits internally. For $x \in (y, \frac{1 - \beta_y}{1 - \beta_x} y)$ the firm now prefers internal ventures, even though spin-offs are more efficient. The employee cannot persuade the firm to do otherwise, since she is wealth constrained.

Consider next relaxing the assumptions $\gamma > \hat{\gamma}_1$ (for Proposition 2) and $\gamma > \hat{\gamma}_2$ (for Proposition 3 and 4). For lower values of γ , the firm is less concerned about focusing employees, since the main benefit of focus occurs when the core prospects are strong. The proofs of Proposition 2 and 3 show that for lower values of γ , the intrapreneurial equilibrium (σ^+, D_y) becomes more prevalent. For $\hat{\gamma}_2 < \gamma < \hat{\gamma}_1$, it replaces the stubborn equilibrium in the *EIP* model. For $\gamma < \hat{\gamma}_2$, it replaces the focused equilibrium, both in the *EIP* and *FIP* model. This reinforces our previous point that as long as the firm assigns the employee a task that is suitable most of the times (high γ), it wants to ensure that the employee remains focused on that task. Yet, if the core task offers poor prospects most of the times (low γ), the firm becomes more open towards idea exploration.

The analysis also uses the assumption $\lambda < \hat{\lambda}$, which says that employees don't get unrelated ideas too often. The condition implies that giving incentives for the core task is expensive for the firm, since the firm has to give bonuses to many employees that do not have any ideas. For larger λ paying a bonus becomes more worthwhile. For $\lambda > \hat{\lambda}$, the firm prefers the "efficient" equilibrium (σ^*, δ^y) . This allows internal ventures, yet prevents excessive exploration through a core bonus ($w_a = pz$). This reinforces the notion that a firm's policy of refusing

to implement internal ventures makes sense only if there are not too many employees that want to generate innovations in the first place.

Finally, it should be mentioned our analysis does rest on two important assumptions. First, we rely on private benefits. For $\beta_y \rightarrow 0$ we obtain $\hat{\lambda} \rightarrow 0$ and the “efficient” equilibrium obtains. The intuition is that without private benefits it is relatively easy for the firm to dissuade the employee from exploring ideas.¹ Second, we rely on the employee’s wealth constraint. Our model builds on Sappington (1983), who introduces a principal-agent model with risk-neutral wealth-constrained agents. Without the wealth constraint, the firm has no real cost to providing incentives for the core task, and the “efficient” equilibrium always obtains. Using wealth constraints has become a popular way of modeling agency costs, because it allows for an intuitive yet tractable model.

¹We believe that private benefits are a natural way of modeling the employee’s preferences. Remember that they include not only non-monetary benefits, but also nontransferable financial returns that the employee can always extract, such as through efficiency wages, information rents or hold-up power. In an earlier version of the paper we explicitly modeled private benefits from hold-up, but then realized that the current specification is both simpler and more general.