Should transactions services be taxed at the same rate as consumption?
Should Transactions Services be Taxed at the Same Rate as Consumption? *

Ben Lockwood† Erez Yerushalmi‡

This version: April 20 2014

Abstract

This paper considers the optimal taxation of transactions services in a dynamic general equilibrium setting, where households use both cash and costly transactions services provided by banks to purchase consumption goods. With a full set of all tax instruments, the optimal tax structure is indeterminate. However, all optimal tax structures distort the relative costs of payment media, by raising the relative cost of deposits to cash. In the simplest optimal tax structure, the Friedman rule holds i.e. cash should be untaxed, and the rate of tax on transactions services can be higher or lower than the consumption tax. When parameters are calibrated to US data, simulations suggest that the transactions services tax should be considerably lower. This is because a transactions tax has a "double distortion": it distorts the choice between payment media, and indirectly taxes consumption. This contrasts with the special case of the cashless economy, when the first distortion is absent: in this case, it is optimal to tax transactions services at the same rate as consumption.

JEL Classification: G21, H21, H25

Keywords: financial intermediation services, tax design, banks, monitoring, payment services

---

*This paper is a substantially revised version of part of CEPR Discussion Paper 8122, "How Should Financial Intermediation Services be Taxed?". I would like to thank Steve Bond, Clemens Fuest, Michael Devereux, Andreas Hauger, Michael McMahon, Miltos Makris, Ruud de Mooij, Carlo Perroni, and seminar participants at the University of Southampton, GREQAM, the 2010 CBT Summer Symposium, and the 2011 IIPF Conference for helpful comments on earlier drafts. I also gratefully acknowledge support from the ESRC grant RES-060-25-0033, "Business, Tax and Welfare".

†CBT, CEPR and Department of Economics, University of Warwick, Coventry CV4 7AL, England; Email: B.Lockwood@warwick.ac.uk

‡Institute for Employment Research, University of Warwick.
1. Introduction

Financial intermediation services include such important services as intermediation between borrowers and lenders, insurance, and payment transaction services (e.g. credit and debit card services). These services comprise a significant and growing part of the national economy; for example, financial intermediation services, measured using the OECD methodology\(^1\), were 3.9% of GDP in the UK in 1970, and increased to 7.9% by 2005. Over the same time period, from 1970 to 2005, the increase for Eurozone countries as a whole has been from 2.7% to 5.5%. In the US, the finance and insurance sector, excluding real estate, which includes financial intermediation, accounted for 7.3% of US value-added in 1999, rising to 8.4% in 2009\(^2\).

The question of whether, and how, financial intermediation services should be taxed is a contentious one\(^3\). Currently, within European Union countries, most financial intermediation services are exempt from VAT, notably financial services which are not explicitly priced. Similarly, in the US, very few states tax financial services\(^4\). Current practice reflects the fact that there are technical difficulties in taxing financial intermediation when those services are not explicitly priced (so-called margin-based services), such as intermediation between borrowers and lenders. However, conceptually, the problems can be solved, for example, by use of a cash-flow VAT (Hoffman et al., 1987; Poddar and English, 1997; Huizinga, 2002; Zee, 2005), and the increasing sophistication of banks’ IT systems means that these solutions are also becoming practical.

As a result, there is considerable debate, at least in Europe, about the possible revenue and welfare gains from imposing VAT on financial intermediation services (de la Feria and Lockwood, 2010; PWC, 2011; Buettner and Erbe, 2012). In this literature, it is largely assumed that within a consumption tax system, such as a VAT, it is desirable to tax...

---

1. This methodology yields FISIM, that is financial intermediation services indirectly measured: see http://www.euklems.net. The latest year for which data are available for EU countries on a consistent basis from the Klems database is 2005: for the UK, data for 2011 show that this share has further risen to 9%.
2. See http://www.bea.gov/industry/gdpbyind_data.htm
3. There are technical difficulties in taxing financial intermediation when those services are not explicitly priced (so-called margin-based services), such as the intermediation between borrowers and lenders. However, conceptually, the problems can be solved, for example, by use of a cash-flow VAT (Hoffman et al., 1987; Poddar and English, 1997; Huizinga, 2002; Zee, 2005), and the increasing sophistication of banks’ IT systems means that these solutions are also becoming practical.
4. Data available from the website of the Federation of Tax Administrators (taxadmin.org) indicates that in 2007, for example, only two states, Indiana and Washington, taxed "service charges of banking institutions".
financial services supplied at the standard rate of VAT, and allow financial intermediation services providers - banks and insurance companies - to claim back VAT they pay on inputs: see e.g. Ebril et al. (2001). Also, the recent IMF proposals for a "bank tax" to cover the cost of government interventions in the banking system include a Financial Activities Tax levied on bank profits and remuneration, one version of which - FAT1 - which would work very much like a VAT (IMF, 2010).

Bringing financial intermediation services within the scope of VAT in this way would (a) tax financial intermediation services to households at the same rates as taxes on other goods; (b) in principle, eliminate existing distortions in the production chain due to the fact that financial intermediation services sales to firms currently include irrecoverable VAT on inputs. Advantage (b) of applying VAT to financial intermediation services is generally accepted. However, it is less clear that the financial intermediation services supplied to households should be taxed at the same rate as other goods and services. This important question has received remarkably little attention. In particular, while there is an established literature on optimal taxation in monetary models, these models without exception, assume either that cash is the only medium of payment, or that some goods can be bought on credit, and so do not address the issue of how intermediation services provided by banks should be taxed. This paper attempts to fill the gap, by study of the optimal tax structure in a setting based on the well-known model of Freeman and Kydland (2000), originally used to study the relationship between money aggregates and output.

In our model, the household demands different varieties of goods in different quantities, and total consumption demand must be met by holdings of cash or bank deposits. The inconvenience of holding large quantities of cash is explicitly modelled, meaning that in equilibrium, there will be a "switch point" above which varieties will be bought using deposits. Competitive banks can provide deposits (and the services associated with them, such as cheques and debit cards) and a cost, and the value-added of banks can be taxed at a rate $\tau_d$. The government can also levy a wage tax (or equivalently a general consumption tax) and an interest income tax, as in the dynamic optimal tax literature. The government then chooses the taxes, plus the rate of inflation, to minimize the deadweight loss of providing a public good in each period.

The solution to this tax design problem yields the following insights. First, the opti-
mal taxes are technically indeterminate, as the government can (implicitly) control four "prices" facing the household; the opportunity cost of cash and of deposits, the price of consumption relative to leisure, and the relative price of present and future consumption. But, the household only makes three choices; consumption, leisure, and the relative proportions of cash and deposits. However, it can be shown that all optimal tax structures distort the relative costs of payment media, by raising the relative cost of deposits to cash.

Moreover, within the class of optimal tax structures, there is only one that is simple and intuitive. This is where the opportunity cost of holding money is zero i.e. the Friedman rule applies. Then, the wage tax (or general consumption tax) is set according to a standard Ramsey formula, and the interest income tax is zero in the steady state, as in Chamley (1986) and Judd (1985).

The tax faced by the households on transaction services (the combination of the general consumption tax and $\tau_d$) bears a simple relationship to the tax on final consumption goods; it depends on the intertemporal elasticity of consumption, the elasticity of labour supply, and the degree of complementarity of goods and leisure via a simple formula. In particular, if the sum of these elements is above (below) a cutoff value, then transactions services should be taxed at a lower (higher) rate than final consumption.

Thus, even when it is optimal to tax all goods at the same rate (justifying a VAT on consumption), payment services are generally taxed at a different rate. We then solve for the steady-state taxes, calibrating model parameters to the US economy. We find that when parameters are calibrated, simulations suggest that the transactions services tax should be considerably lower, specifically, around one sixth of the consumption tax. The intuition is that a transactions tax has a "double distortion": it distorts the choice between payment media, and indirectly taxes consumption. By contrast, the consumption tax does not distort the choice between payment media, and so it is typically set at a higher level.

This finding has implications for the current policy debate on the taxation of banks, especially in Europe, where it is view of many, including the European Commission, that banks are undertaxed, because many of their services are exempt from VAT. Our results imply that this particular form of under-taxation may not be of great concern. Of course, there are other reasons for taxing banks, for example, to charge ex ante for the social costs of bailouts, or corrective taxes to discourage excessive risk-taking, and so on.

Finally, our finding of different taxes on transactions and consumption contrasts with an important earlier finding in the literature, Auerbach-Gordon (2002). They consider a life-cycle model of the consumer where purchase of goods requires transactions services, which are assumed to be demanded in strict proportion to consumption. They show
that if there is initially only a labour income tax imposed on the household, then this is equivalent to a value-added tax if and only if the transactions services consumed by the household are taxed at the same rate as other goods. This finding is often taken to justify taxing transactions services at the same rate as consumption. The Auerbach-Gordon result can be reconciled with our results by noting that theirs is a cashless economy. In our model, if cash is not available, the optimal tax on transactions services is then not independently determined, and a tax at the same rate as consumption is an optimal tax structure. The intuition here is that when cash is not available, a transactions tax cannot distort the choice of payment medium, and so there is no longer a "double distortion".

The remainder of the paper is organized as follows. Section 2 discusses related literature. Section 3 outlines the model, and Section 4 presents the main results. Section 5 studies a calibrated version of the model, and Section 6 concludes.

2. Related Literature

Our paper relates to several literatures. First, our model is a variant of the model of cash and bank deposits due to Aiyagari, Braun, and Eckstein (1998) and further developed by English (1999), Freeman and Kydland (2000), and Henriksen and Kydland (2010), amongst others. In this class of models, the relative size of demand for different varieties of the consumption good is fixed by a fixed coefficients utility index such as (3.2), and then the use of cash or deposits to buy a particular variety \( j \) is determined by the relative fixed costs of using the two media. The initial purpose of these papers is to study the effect of inflation on the volume of transactions services (bank deposits and credit cards) provided by the banking sector; it is well documented that the use of transactions services rises when inflation rises. Subsequently, Freeman and Kydland (2000) and Šustek (2010) have used this type of model to explain various stylized facts about the co-movement of various money aggregates and output. Because these models are fully micro-founded, they are ideal for the study of the optimal tax on transactions services. Our paper is the first, to our knowledge, to fully analyze the optimal tax structure in this important class of models\(^6\).

The second related literature is on dynamic optimal taxation, in particular, that part of the literature focussed on the optimal inflation tax\(^7\). This literature, building on the

---

\(^6\) Henriksen and Kydland (2010) compare the marginal cost of public funds from an inflation tax to that from a labour tax, but they do not consider the taxation of transactions services.

\(^7\)We also assume linear income taxes, full commitment, and no information asymmetries, assumptions that are shared by most papers on the optimal inflation tax.
seminal contributions of Chamley (1986) and Judd (1985) is of course, large. However, as mentioned in the introduction, these models without exception, either assume (i) that cash is the only medium of payment, or (ii) assume that an exogenously specified subset of goods can be bought on credit, and are thus not subject to a cash-in-advance constraint. So, these contributions do not address the issue of how intermediation services provided by banks should be taxed. The most closely related contributions are Chari et al. (1991, 1996), who show that in a cash-in-advance model with credit goods, the optimal inflation tax is zero if utility is separable in consumption goods and leisure, and the consumption sub-utility function is homothetic. This is related to our Proposition 1 below. Bhattacharya et al. (2005) explore the optimality of the Friedman rule in a similar model with two-period lived consumers. These papers, however, do not allow for a banking sector or costly transactions.\textsuperscript{8}

Thirdly, there is a small literature directly addressing the optimal taxation of borrower-lender intermediation and payment services (Grubert and Mackie, 1999; Jack, 1999; Auerbach and Gordon, 2002; and Boadway and Keen, 2003). With the exception of Auerbach and Gordon (2002) - henceforth AG - these papers use a simple two-period consumption-savings model without an explicit production sector, and assume that payment services are consumed in fixed proportion to aggregate consumption\textsuperscript{9}. In this setting, it is straightforward to show that if there is a pre-existing consumption tax at the same rate in both periods, the marginal rate of substitution between present and future consumption is left unchanged if payment services are taxed at the same rate as consumption. AG consider a multi-period life-cycle model of the consumer where purchase of goods requires transactions services, which themselves are produced using other inputs. Transactions services are assumed to be demanded in strict proportion to consumption. They show that if there is initially only a labour income tax imposed on the household, then this is equivalent to a value-added tax if and only if the transactions services consumed by the household are taxed at the same rate as other goods\textsuperscript{10}.

There are, however, a number of restrictive assumptions implicit in these existing

---

\textsuperscript{8}There is also a less closely related literature which studies the optimal inflation tax with a transaction costs approach to the demand for money (Kimbrough, 1986; Guidotti and Vegh, 1993; Correia and Teles, 1996, 1999). They also find optimality of the Friedman rule under certain conditions.

\textsuperscript{9}Chia and Whalley (1999), using a computational approach, reach the rather different conclusion that no intermediation services should be taxed, but their model is not directly comparable to these others, as the intermediation costs are assemmed to be proportional to the price of the goods being transacted.

\textsuperscript{10}In particular, they show that if there is initially a wage income tax at rate $\tau$, which is replaced by a consumption tax at equivalent rate $\tau/(1 - \tau)$, then the real equilibrium is left unchanged if and only if transaction services are also taxed at this equivalent rate.
models. First, other taxes are assumed fixed, not optimized, and it is implicit that the existing taxes are non-distortionary, because the analysis proceeds by finding conditions under which taxation of transaction services does not introduce any further distortions. In turn, the only way in which a uniform consumption tax (or equivalently, a wage income tax) can be non-distortionary is if labour is in fixed supply, so it is arguable that this is a further implicit assumption of the above studies. By contrast, we take an explicit tax design approach to the question, assuming a household demand for leisure, and investigating the second-best tax structure, given that there is a revenue constraint.

Second, and equally importantly, one can argue that the modelling of transactions services in the existing literature is at an abstract level, and not microfounded in any way; the papers above simply assume that the cost of these services is proportional to consumption. This corresponds to a special case of our model where cash is prohibitively expensive, so the tax system cannot affect the choice of payment medium. In that special case, we find that taxing consumption and transaction services at the same rate is optimal, consistently with this existing literature.

Finally, there has recently been surge of literature\textsuperscript{11} studying banks that engage in socially undesirable activities such as excessive risk-taking on both lending and deposit-taking margins. The main finding is that these should be corrected by Pigouvian taxes (or regulations) that apply directly to these decision margins, such as taxes on borrowing or lending, not by making VAT on bank inputs irrecoverable. Our work is distinct from this line of enquiry, as bank lending has no external effects in our setting; we are concerned with the design of taxes to raise revenue.

3. The Model

3.1. Firms

In each period $t = 0, \ldots, \infty$, a single competitive firm produces an intermediate good from labour and capital via the production function $f(k_t, h_t)$, where $k_t$ is the capital stock, and $h_t$ is hours of work supplied by the household. One unit of this intermediate good can be transformed into one unit of a continuum of different varieties $j \in [0, 1]$ of a consumption good, an investment good, a public good; and also into $1/\psi$ units of banking services. The nature of banking services is discussed in 3.2 below. The assumption that labour is not needed to produce final goods, the investment good and banking services is

for convenience only and could be relaxed at the cost of additional complexity, without changing the main results. Capital depreciates at rate $\delta$, so follows the usual process:

$$k_{t+1} = i_t + (1 - \delta)k_t. \quad (3.1)$$

Capital is rented from households and banks, at rental rate $r_t = f_{kt} - \delta$. The wage is determined by the usual condition $w_t = f_{ht}$. We use (here and below) the notation that for any function $f(x_t, y_t)$, the partial derivative of $f$ with respect to $x_t$ is $f_{xt}$, the cross-derivative is $f_{xyt}$ etc.

### 3.2. Households

There is a single infinitely lived household with preferences over levels of consumption goods, leisure, and a public good in each period $t = 0, \ldots, \infty$ of the form

$$\sum_{t=0}^{\infty} \beta^t(u(c_t, l_t) + v(g_t)), \quad c_t = \min_{j \in [0, 1]} \{c_t(j)/2j\} \quad (3.2)$$

where $c_t(j)$ is the level of consumption of variety $j$ in period $t$, $l_t$ is the consumption of leisure, and $g_t$ is public good provision. Utilities $u(c, l)$, $v(g)$ are strictly increasing and strictly concave in their arguments and $0 < \beta < 1$ is a discount factor. We also assume $u_{cl} \geq 0$, an assumption further discussed below.

The fixed coefficients specification for the commodity index follows Freeman and Kydland (2000); it allows for consumption levels of the different varieties to vary in an analytically tractable way. Specifically, given an aggregate level of consumption, $c_t$, it is optimal to set consumption of variety $j$ at $c_t(j) = 2j c_t$, so consumption of variety $j$ is increasing in $j$.

Following Englund and Svensson (1988), and Henriksen and Kydland (2010), we assume that the household faces a generalized cash-in-advance constraint; in period $t$, the total nominal value of consumption must be matched by beginning of period holdings of either cash ($M_t$) or demand deposits ($D_t$).\footnote{Often, these holdings are dated $t-1$, e.g. Walsh (2003), but it is more convenient to date them by $t$ here.} There is a substantial empirical literature on the use of cash versus other payment media, such as debit cards (Snellman et al., 2001; Lippi and Secchi, 2009; Ten Raa and Shestalova, 2004). This literature finds that the choice between the two is determined by: (i) the relative opportunity cost of the two media; (ii) fees for the use of electronic payment media, and (iii) non-pecuniary costs, such as time and inconvenience; (iv) the risk of having cash lost or stolen. Opportunity costs...
alone would imply a corner solution where only cash or electronic media are used. This is inconsistent with what is observed in practice, where cash is used for small transactions, and cards for larger transactions\textsuperscript{13}.

To model this, we suppose that cash becomes increasingly costly for large transactions, via the increasing inconvenience of carrying it and keeping it safe, whereas the cost of using deposits will be proportional to transactions. Formally, we suppose that there is a fixed time cost $\gamma j^\sigma$, $\sigma > 1$, incurred by the household if variety $j$ is bought with cash.

As $\sigma > 1$, the marginal cost of buying with cash rises faster than the marginal benefit of buying with cash, which is positive and linear in $j$ when the opportunity cost of cash (defined below) is lower than that of deposits\textsuperscript{14}. This implies a cutoff $j^*_t$ such that only goods $j \leq j^*_t$ will be bought with cash. Then the transactions constraints facing the household can be written

\begin{equation}
\frac{M_t}{P_t} \geq \int_0^{j^*_t} c_t(j) dj = K^m_t c_t, \quad \frac{D_t}{P_t} \geq \int_{j^*_t}^1 c_t(j) dj = K^d_t c_t, \tag{3.3}
\end{equation}

where $P_t$ is the price of the intermediate good. Note that $K^d_t + K^m_t = 1$. The first of these is just a cash-in-advance constraint, and the second is similar, in that it requires that real deposits must be no less than the real value of goods purchased using bank deposits. We refer to these two constraints collectively as \textit{transactions constraints}.

In each period, the household consumes goods, supplies leisure, and can accumulate capital, cash, or deposits. It must also divide its total time endowment between leisure, work, and time lost to transacting in cash, so

\begin{equation}
h_t = 1 - l_t - \gamma \int_0^{j^*_t} j^\sigma dj \equiv 1 - l_t - \phi(j^*_t) \tag{3.4}
\end{equation}

where $\phi(j^*_t) = \frac{\gamma j^*_t^{\sigma+1}}{\sigma+1}$ is the total time cost incurred in dealing with cash purchases for varieties $0 \leq j \leq j^*_t$. The nominal budget constraint says that the cost of consumption, $c_t P_t$, plus end of period holdings of cash and deposits $M_{t+1}, D_{t+1}$, plus purchases of the capital good by the household, $P_t k^H_{t+1}$, must be equal to wage income plus nominal gross

\textsuperscript{13}For example, using a sample of Dutch retailers, Ten Raa and Shestalova (2004) estimate that the point at which households switch from cash to electronic payment media is somewhere between 13 and 30 Euros.

\textsuperscript{14}In the case where the opportunity cost of cash is higher, no goods are bought with cash i.e. $j^*_t = 0$. 

returns on holdings of money, deposits, and capital:

\[ c_t P_t + M_{t+1} + D_{t+1} + P_t k_t^H = \]

\[ w_t P_t (1 - \tau^w_t) h_t + M_t + (1 + \tilde{r}_t (1 - \tau^r_t))(1 + \pi_t) \]

\[ d_t + (1 + r_t (1 - \tau^r_t))(1 + \pi_t) P_{t-1} k_t^H \]

where \( \pi_t = \frac{p_t}{P_{t-1}} - 1 \), and \( \tau^r_t \) is an interest income tax, \( \tau^w_t \) is a wage income tax. Moreover, \( \tilde{r}_t \) is the real pre-tax return paid by banks on deposits, determined below, so \( (1 + \tilde{r}_t (1 - \tau^r_t))(1 + \pi_t) \) is the nominal gross post-tax return on deposits. In the same way, \( (1 + r_t (1 - \tau^r_t))(1 + \pi_t) \) is the nominal gross post-tax return on capital.

Dividing through by \( P_t \), we can write this per period budget constraint in real terms as follows:

\[ c_t + m_{t+1} (1 + \pi_{t+1}) + d_{t+1} (1 + \pi_{t+1}) + k_t^H = \]

\[ w_t (1 - \tau^w_t) h_t + m_t + (1 + \tilde{r}_t (1 - \tau^r_t))(1 + \pi_t) d_t + (1 + r_t (1 - \tau^r_t)) k_t^H, \]

\[ t = 1, 2, \ldots \]

Finally, following Chari et al. (1996), we assume that \( M_0 = D_0 = k_0^H = 0 \); if these initial conditions do not hold, then the government’s problem is trivial.\(^{15}\)

Eliminating the \( k_t^H \) by successive substitution in (3.5), using the fact that \( m_0 = d_0 = k_0^H = 0 \), bringing all terms in \( m_t, d_t \) to the left-hand side and using (3.4), the present-value budget constraint can eventually be written

\[ \sum_{t=1}^{\infty} R_t (c_t + \psi^{m}_t m_t + \psi^{d}_t d_t) = \sum_{t=1}^{\infty} R_t (w_t (1 - \tau^w_t)(1 - l_t - \phi(j^*_t))) \]

(3.6)

where

\[ R_t = \prod_{j=1}^{t} \frac{1}{1 + r_j (1 - \tau^r_j)}, \quad \psi^{m}_t = (1 + \pi_t)(1 + r_t (1 - \tau^r_t)) - 1, \quad \psi^{d}_t = (1 + \pi_t)(1 - \tau^r_t)(r_t - \tilde{r}_t) \]

(3.7)

So, \( \psi^{m}_t, \psi^{d}_t \) are the opportunity costs of holding cash and deposits relative to capital.

The household chooses \( \{c_t, l_t, m_t, d_t, j^*_t\}^{\infty}_{t=1} \), to maximize \( \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \) subject to (3.6), (3.4), (3.3). Without loss of generality, we can assume the transactions constraints (3.3) hold with equality i.e. \( m_t = K^{m}_t c_t, \quad d_t = K^{d}_t c_t \). Then, we can substitute out \( m_t, d_t \) in (3.6) and just optimize with respect to \( \{c_t, j^*_t, \quad l_t\}^{\infty}_{t=1} \). The first-order conditions are:

\[ c_t : \beta^t u_{ct} = \lambda R_t (1 + \psi^{m}_t K^{m}_t + \psi^{d}_t K^{d}_t) \]

(3.8)

\[ l_t : \beta^t u_{lt} = \lambda w_t (1 - \tau^w_t) R_t \]

(3.9)

\[ j^*_t : w_t (1 - \tau^w_t) \gamma (j^*_t)^{\gamma} = c_t j^*_t (\psi^{d}_t - \psi^{m}_t) \]

(3.10)

\(^{15}\)As is well-known, if the initial stock \( M_0 + D_0 \) of nominal assets is positive (negative), then welfare is maximized by setting the initial price level to infinity (or sufficiently low). See Chari et al. (1996), p207.
where $\lambda$ is the multiplier on (3.6), and in the last line, we assume an interior solution $0 < j^*_t < 1^{16}$. Condition (3.8) says that the "full" price of consumption comprises the purchase price $1$, plus the cost of transacting, namely $\psi^m_t K^m_t + \psi^d_t K^d_t$. Condition (3.10) says that at an interior solution, the optimal choice of $j^*_t$ balances the lower opportunity cost of holding cash, equal to $j^*_t (\psi^d_t - \psi^m_t)$, against the additional inconvenience cost of cash i.e. $w_t (1 - \tau^w_t) \gamma (j^*_t)^\sigma$. It can be solved explicitly to yield $j^*_t = \left( \frac{(\psi^d_t - \psi^m_t) \gamma_t}{\gamma_t w_t (1 - \tau^w_t)} \right)^1 / (\sigma - 1)$. Corner solutions for $j^*_t$ can be characterized in the obvious way. Below, we characterize optimal taxes under the realistic assumption that at the second-best optimum, $j^*_t$ is interior i.e. households use both cash and deposits.

### 3.3. Banks

There are a large number of competitive banks who provide demand deposits to households, and use the funds to purchase capital. Without loss of generality, there is no reserve requirement, so the bank balance sheet in real terms can be written $\frac{D_t}{P_t} = k_t^B$, where $k_t^B$ is the bank holding of capital in period $t$. We have assumed above that payment services (e.g. debit cards) associated with a unit of real deposits require $\psi$ units of the intermediate good and so the nominal cost of providing the payment services associated with $D_t$ is $\frac{D_t}{P_t} \psi P_t = \psi$. The difference in nominal returns between capital and deposits i.e. $(1 + \pi_t) (r_t - \tilde{r}_t)$ is the value-added of the bank per unit of deposit, and is taxed at rate $\tau^d_t$. Then, as banks make zero profit, the after-tax value added per unit of $D_t$ must be equal to the cost of payment services per unit of $D_t$ i.e.

$$\frac{(1 + \pi_t) (r_t - \tilde{r}_t)}{(1 + \tau^d_t)} = \psi$$

Finally, note from (3.11), (3.7) that

$$\psi^d_t = \psi (1 - \tau^d_t) (1 + \tau^d_t) \equiv \psi (1 + \tilde{\tau}^d_t)$$

So, ultimately, the opportunity cost of holding deposits for the household is $\psi$, grossed up by the effective tax on banking services, $\tilde{\tau}^d_t$.

### 3.4. Government

The government finances the public good $g_t$ from tax revenues generated from taxes $\tau^w_t, \tau^r_t, \tau^d_t$ and also seigniorage revenues $m_{t+1} (1 + \pi_t) - m_t$. We do not specify the

---

16This requires that the marginal cost of cash at $j^* = 1$ exceeds the marginal benefit i.e. $\gamma > c_t (\psi^d_t - \psi^m_t)$.  

11
nominal money growth rule at this point; as we will see below, at the second-best optimum, nominal money growth will be chosen so as to implement the Friedman rule, that the real opportunity cost of money, $\psi_t^m$, should be zero. We do not need to explicitly write out the government budget constraint, as we use the primal approach to the tax design problem, as explained below. Rather, the government faces the resource constraint for the economy, which is

$$\int_0^1 c_t(j) dj + k_{t+1} - (1 - \delta)k_t + g_t + \psi d_{t+1} \leq f(k_t, h_t)$$

(3.13)

where $g_t$ is the quantity of the public good required at time $t$.

### 3.5. Discussion

This model provides a general framework which encompasses\textsuperscript{17} the specific models of taxation of payment services (AG; Boadway and Keen, 2003; Jack, 1999; Grubert and Mackie, 1999) that have been developed so far. Indeed, if one removes cash from the model, i.e. set $j_t^* = 0$, we see that $K^d_t = 1$, and thus the overall price of $c_t$, excluding all taxes is $1 + \psi$; i.e. there is a direct cost of 1 unit of the intermediate good, plus an additional fixed transaction cost of $\psi$. This compares to AG, who assume that the consumer price of good $i$ at time $t$ is $p_{it} = c_{it} + b_{it}$, where $c_{it}$ is the production cost of the good, and $b_{it}$ captures any transactions cost of actually acquiring the good”. This suggests that “microfoundations” for the AG model can be given in this setting by assuming that cash is prohibitively costly.

Our model is also very close to the well-known Freeman and Kydland (2000) model. The key difference is that in their model, a non-trivial choice between cash and bank deposits is achieved by introducing a fixed cost of deposits (specifically, a fixed cost of paying for variety $j$), whereas in ours, it is generated by a time cost of using cash that is increasing and convex in $j$. Our reason for departing from the Freeman and Kydland specification is simply that at the second-best optimum, the Friedman rule holds i.e. the opportunity cost of cash is zero, and so with an additional cost of using deposits, at the optimum, the household would always be at a corner, using only cash. But then, the optimal tax on banking services would be undefined. It seems to us that both assumptions are empirically plausible: there is undoubtedly some cost of setting up a bank account, but at the same time, carrying large amounts of cash is inconvenient and risky.

\textsuperscript{17}These papers also allow for savings intermediation, which can be taxed. The principles determining the tax on this spread are somewhat different, and are analysed in a separate paper, Lockwood (2014).
4. Optimal Tax Rules

We take a primal approach to the tax design problem. In this approach, an optimal policy for the government is a choice of all the primal variables in the model, in this case \( \{c_t, l_t, m_t, d_t, j_t^*, k_t, g_t\}_{t=0}^{\infty} \) to maximize utility (3.2) subject to aggregate resource, and implementability constraints. The latter ensures that the government’s choice is consistent with household utility maximization, and it is obtained by substituting the household first-order conditions into the budget constraint. Substituting (3.8), (3.9) into (3.6), and rearranging, we get:

\[
\sum_{t=1}^{\infty} \beta^t (u_t c_t - u_{lt} (1 - l_t - \phi(j_t^*))) = 0 \quad (4.1)
\]

which is the implementability constraint.

Also, the government must take into account the transactions constraints (3.3). Finally, the government also faces (3.13), (3.1), and (3.4). We combine these three to get a resource constraint of the form

\[
c_t + k_{t+1} - (1 - \delta)k_t + g_t + \psi d_t \leq f (k_t, 1 - l_t - \phi(j_t^*)) \quad (4.2)
\]

We now turn to the government’s objective, which is

\[
\sum_{t=0}^{\infty} \beta^t (u(c_t, l_t) + v(g_t))
\]

As is standard in the primal approach to tax design, we can incorporate the implementability constraint (4.1) into the government’s maximand by writing an effective objective for the government of

\[
W_t = u(c_t, l_t) + v(g_t) + \mu (u_{ct} c_t - u_{lt} (1 - l_t - \phi(j_t^*))) \quad (4.3)
\]

where \( \mu \) is the Lagrange multiplier on (4.1). As we have assumed that \( u_{ct} \geq 0 \), i.e. consumption and leisure are complements, it is possible to show that \( \mu \geq 0 \) at the solution to this tax design problem (see Appendix). If \( \mu = 0 \), the revenue from profit taxation is sufficient to fund the public good, \( g \). We will rule out this uninteresting case, and so will assume that \( \mu > 0 \) at the optimum in what follows.

Also, note for future reference that

\[
W_{ct} = u_{ct} (1 + \mu(1 + H_{ct})), \quad W_{lt} = u_{lt} (1 + \mu(1 + H_{lt})) \quad (4.4)
\]
where $H_{ct}$ and $H_{lt}$ are defined by

\begin{align}
H_{ct} &= \frac{1}{u_{ct}} (u_{ct}c_t - u_{ct}(1 - l_t - \phi(j^*_t))) \\
H_{lt} &= \frac{1}{u_{lt}} (u_{ct}c_t - u_{lt}(1 - l_t - \phi(j^*_t)))
\end{align}

Here, $u_{ct}$ etc. denote cross-partial of $u$ with respect to $c_t, l_t$. So, $H_{ct}$ is what Atkeson et al. (1999) call the general equilibrium expenditure elasticity. Note that if there are no transactions costs, i.e. $\gamma = 0$, $H_{lt}, H_{ct}$ reduce to standard formulae found, for example, in the primal approach to the static tax design problem (Atkinson and Stiglitz, 1980).

The Lagrangean for the government’s tax design problem is:

\begin{align}
L &= \beta' u(c_t, l_t) + v(g_t) + \mu (u_{ct}c_t - u_{lt}(1 - l_t - \phi(j^*_t))) \\
&\quad+ \xi_t (f(k_t, 1 - l_t - \phi(j^*_t)) - c_t - k_{t+1} + (1 - \delta)k_t - g_t - \psi d_t) \\
&\quad+ \xi^m_t (m_t - c_tK^m_t) + \xi^d_t (d_t - c_tK^d_t)
\end{align}

where $\xi_t, \xi^d_t, \xi^m_t$ are the multipliers on (4.2) and the transactions constraints (3.3) respectively. The first-order conditions are:

\begin{align}
c_t : \beta' W_{ct} &= \xi_t + \xi^d_t K^d_t + \xi^m_t K^m_t \\
l_t : \beta' W_{lt} &= f_{lt}\xi_t \\
d_t : \xi^d_t &= \xi_t \psi \\
m_t : \xi^m_t &= 0 \\
j^*_t : (\beta' u_{lt} - f_{lt}\xi_t) \phi'(j^*_t) - \xi^m_t c_tj^*_t + \xi^d_t c_tj^*_t &= 0 \\
k_t : \xi_t(f_{kt} + 1 - \delta) - \xi_{t-1} &= 0 \\
g_t : \beta' v_{gt} - \xi_t &= 0
\end{align}

The starting point for the analysis of these conditions is to observe that there is no unique optimal tax structure. The intuition is that the government can (implicitly) control four "prices" facing the household; the opportunity cost of cash and of deposits, the price of consumption relative to leisure, and the relative price of present and future consumption. But, the household only makes three choices; consumption, leisure, and the relative proportions of cash and deposits (i.e. $j^*_t$ : total real cash and deposits holdings at the beginning of the period are constrained to be equal to the value of consumption via (3.3)).

However, it is possible to establish a general property of all optimal tax structures on the two payment media, cash and deposits. The true opportunity cost of cash is zero,
and given a fixed wage tax, the true opportunity cost of deposits in units of labour is \( \psi/w_t \). On the other hand, the difference between the cost of deposits and cash to the household in units of labour is \( (\psi^d_t - \psi^m_t)/w_t(1 - \tau^w_t) \); the wage tax increases the cost to the household of any good or service. We can then show\(^{18}\):

**Proposition 1.** All optimal tax structures raise the relative cost of deposits to cash. Specifically, in units of labour, the difference between the cost of deposits and cash to the household, \( (\psi^d_t - \psi^m_t)/w_t(1 - \tau^w_t) \), should exceed the difference in the true opportunity cost, \( \psi/w_t \).

The intuition for this result is the following. From (4.7), an increase in the use of cash (an increase in \( j^*_t \)) increases maximum welfare for the government by amount \( \mu u_t \phi(j^*_t) > 0 \), ultimately because it relaxes the implementability constraint.

To proceed, we focus on one and only one optimal tax structure that is simple and intuitive. This involves setting the opportunity cost of cash equal to zero, i.e. \( \psi^m_t = 0 \) i.e. leaving cash untaxed. In turn, this implies \( 1 + \pi_t = (1 + r_t(1 - \tau^c_t))^{-1} < 0 \) i.e. deflation just offsets the real return on capital to makes the cost of cash equal to zero. This is of course, the Friedman rule. Given this, a simple and intuitive formula for the tax on deposits emerges.

To state this formula, which involves the wage tax \( \tau^w_t \), it is also convenient to transform \( \tau^w_t \) to the equivalent tax on all consumption \( \tau^c_t \), including consumption of payment services, by observing that \( 1 + \tau^c_t \equiv 1/(1 - \tau^w_t) \). Using \( \psi^m_t = 0 \) and \( 1 + \tau^c_t = 1/(1 - \tau^w_t) \) in the household budget constraint (3.6), we have

\[
\sum_{t=1}^{\infty} R_t (c_t (1 + \tau^c_t) + d_t \psi(1 + \tau^d_t)(1 + \tau^c_t)) = \sum_{t=1}^{\infty} R_t (w_t (1 - l_t - \phi(j^*_t)))
\]

(4.15)

So, from (4.15), it is clear that payment services are taxed at overall rate:

\[
\hat{\tau}^d_t = (1 + \tau^c_t)(1 + \hat{\tau}^d) - 1
\]

We can then state an optimal rule for \( \hat{\tau}^d_t \):

**Proposition 2.** At any date \( t \), the following taxes on cash and deposits are optimal. First, the Friedman rule \( \psi^m_t = 0 \) holds. Second, the effective tax on deposits in ad valorem form is:

\[
\frac{\hat{\tau}^d_t}{1 + \hat{\tau}^d_t} = \left( \frac{v_{gt} - u_{lt}/w_t}{v_{gt}} \right) \frac{1}{1 + H_{lt}}
\]

\(^{18}\)All proofs (where required) are in the Appendix.
The key part of this Proposition is (4.16), which characterizes the optimal effective tax on payment services. To interpret (4.16), consider first the first term on the right-hand side. If the household supplied one more unit of labour, this would cost $u_{lt}$ in forgone utility, but could be used to produce $w_t$ more units of the public good, which the household values at $v_{gt}$. So, $\frac{u_{lt} - w_t}{v_{gt}}$ is a measure of the social gain from additional taxation at the margin, and we assume this is positive. To interpret the second term on the right-hand side, note first that given our assumption $u_{cl} \geq 0$, $H_{lt}$ is strictly positive from (4.6), so $\nu^d_t$ is strictly positive. So, as we would expect from Proposition 1, deposits are taxed "more heavily" than cash - the former at a strictly positive rate, the latter at zero.

Finally, in the simple case where $u_{cl} = 0$, $H_{lt}$ reduces to the elasticity of utility with respect to leisure, times $h_l/l_t$. So, roughly speaking, $H_{lt}$ measures the elasticity of labour supply. This is an exact statement when in addition, $u$ is linear in $c$. So, we conclude that the tax on deposits becomes higher when (i) the value of taxation at the margin is higher, and (ii) when the elasticity of labour supply is lower.

Our result on the Friedman rule is, by contrast, less original. For example, Henriksen and Kydland (2010) note that in their version of the Aiyagari model, the Friedman rule is optimal. Our analysis does show, however, that in a fully specified tax design problem with a full set of tax instruments, the Friedman rule is not uniquely optimal. Proposition 1 is also related to Chari et al. (1991, 1996) who show that in a cash-in-advance model with credit goods, the optimal inflation tax is zero if utility is separable in consumption goods and leisure, and the consumption sub-utility function is homothetic. In our setting, these conditions are in fact satisfied, because $\min_j c_t(j)/j$ is a homothetic sub-utility function. Of course, in our model, there are no credit goods; rather the purchase of some goods is financed from bank deposits.

We now turn to the key question of whether payment services should be taxed at a higher or lower rate than consumption i.e. whether $\nu^d_t$ exceeds $\tau^c_t$ or not. As a first step, we can obtain the following characterization of the optimal effective total tax rate on consumption:

---

$19$ Note that generally, the government can implement the Friedman rule $\psi_t^n = 0$ by appropriate choice of money growth rule. To see this, note that if the nominal money stock grows at $\sigma_t$, the real money stock grows at rate $\Delta m_t = \frac{1 + \sigma_t}{1 + r_t} - 1$. So, to achieve the Friedman rule, $\sigma_t$ can be set so that $\sigma_t = \frac{\Delta m_t}{m_t} + \frac{1}{1 + r_t (1 - \tau^c_t)} - 1$. 

---
Proposition 3. At any date $t$, the optimal effective total tax rate on consumption is

$$\frac{\tau_t^c + \psi K^d_t \tilde{\tau}_t}{1 + \tau_t^c + \psi K^d_t (1 + \tilde{\tau}_t)} = \left( \frac{v_{gt} - u_{lt}/u_t}{v_{gt}} \right) \left( \frac{H_{lt} - H_{ct}}{1 + H_{lt}} \right)$$

where $H_{ct}, H_{lt}$ are defined in (4.5), (4.6).

On the left-hand side of (4.17), we have the overall effective tax rate on consumption, $\tau_t^c + \psi K^d_t \tilde{\tau}_t$, in ad valorem form. The interpretation is that one unit of final consumption requires one unit of the marketed good, taxed at $\tau_t^c$, and (conditional on the optimal choice of $j_t^c$), $\psi K^d_t$ units of payment services.

Note that the right-hand side of (4.17) is identical to the formula for an optimal consumption tax that also occurs in the static optimal tax problem, when the primal approach is used (Atkinson and Stiglitz 1980, p377). First, as above, $\frac{v_{gt} - u_{lt}/u_t}{v_{gt}}$ is a measure of the social gain from additional taxation at the margin, and $H_{lt}$ measures the elasticity of labour supply. Second, by inspection of (4.4), $-H_{ct}$ measures the degree of complementarity between consumption and leisure; the higher this is, other things equal, the higher the total effective tax on consumption, a well-known result.

Finally, we are now in a position to address the question of the relative size of $\tilde{\tau}_t$ and $\tau_t^c$, which is the central focus of this paper. Note that the left-hand side of (4.17) is the weighted combination of $\frac{\tilde{\tau}_t}{1 + \tilde{\tau}_t}$ and $\frac{\tau_t^c}{1 + \tau_t^c}$, and that the right-hand sides of (4.16),(4.17) differ only by the factor $H_{lt} - H_{ct}$. The following result is then immediate:

Proposition 4. (a) If $H_{lt} - H_{ct} = 1$, then $\tilde{\tau}_t = \tau_t^c$; (b) if $H_{lt} - H_{ct} > 1$, $\tilde{\tau}_t < \tau_t^c$; (c) if $H_{lt} - H_{ct} < 1$, $\tilde{\tau}_t > \tau_t^c$.

To interpret this, consider a steady state to lighten notation, and then recall from (4.5), (4.6) that

$$H_l - H_c = \frac{u_{wd} c}{u_c} + \frac{u_{ld} h}{u_l} + u_{cl} \left( \frac{c}{u_l} + \frac{h}{u_c} \right)$$

(4.18)

So, other things equal, the higher the complementarity of consumption and leisure i.e. the higher $u_{cd}$, the higher is $H_l - H_c$, and thus the more likely it is that consumption is taxed more highly than transactions. In the same way, as $\frac{u_{wc} c}{u_c}$ is an inverse measure of the elasticity of consumption, the less elastic is demand for consumption, more likely it is that consumption is taxed more highly. Finally, as $\frac{u_{wd} h}{u_l}$ is an inverse measure of the

\footnote{However, inspection of (4.5) and (4.6) reveals that in our analysis, the $H_{lt}, H_{ct}$ are generally different to the static case because of the term $\phi(j_t^c)$.}
elasticity of labour supply, the the less elastic is demand for labour, more likely it is that consumption is taxed more highly.

The intuition for these findings is as follows. Note that the transaction tax creates a double distortion; it distorts \( j_t^* \), the choice of payment medium, and via its effect on the left-hand side of (4.17), it contributes to the overall tax on consumption, and thus the consumption-leisure choice of the household. On the other hand, the consumption tax only distorts the second of these decisions. So, the transaction tax is relatively more distorting when deadweight loss from a consumption tax is relatively low. This in turn is the case when the consumption and leisure (or labour) choices of the household are relatively inelastic, or when consumption and leisure are very complementary.

To get a feel for the overall size of (4.18), note that a standard specification in the macroeconomics literature would be to take both \( \frac{\mu_{cc}}{u_c} \) and \( \frac{\mu_{ul}}{u_l} \) to be around 2.5. Moreover, for the US, working hours are about 1/3 of the total time allocation, implying an \( h/l \) of about 1/2. Thus, even without any consumption-leisure complementarity, \( H_I - H_c \) would certainly be greater than 1, and thus \( \hat{\tau}^d < \tau^c \). Detailed calibrations reported in the next Section suggest that in fact, \( \hat{\tau}^d \) is about one-fifth of \( \tau^c \) on average, when parameters are randomly sampled from distributions centred on their calibrated values.

We can also reconcile these results with the earlier literature on taxation of transactions services discussed in Section 2. As argued above in Section 3.5, a special case of our model where cash is not available as a payment medium is compatible with AG. This can be captured formally by setting \( j_t^* = 0 \) by definition; thus the first-order condition (4.12) does not apply. This in turn implies that condition (4.16) no longer applies. In this case, the optimal taxes \( \tau_t^c, \hat{\tau}_t^d \) are only characterized by (4.17). It is then clear from that there is an additional indeterminacy; there are an infinite number of combinations of \( \tau_t^c, \hat{\tau}_t^d \) that satisfy (4.17). However, it is clear from (4.17) that one possible optimal structure is where transactions and final consumption are taxed at the same rate i.e. \( \tau_t^c = \hat{\tau}_t^d \). This is of course, the same finding as AG. We can summarize as follows:

**Proposition 5.** In the special case of the cashless economy, optimal taxes \( \tau_t^c, \hat{\tau}_t^d \) are indeterminate, but one optimal tax structure is equal taxation of transactions and final consumption i.e. \( \tau_t^c = \hat{\tau}_t^d \).

We complete our analysis of the tax rules by considering the capital income tax, \( \tau_t^r \). Here, we can show that in the steady state, the Chamley-Judd result holds in our model, i.e. \( \tau_t^r = 0 \).

**Proposition 6.** In the steady state, \( \tau_t^r = 0 \).
This simplifies the implementation of the tax rules $\tau^c$, $\tau^d$; in the steady state, they can be achieved just by two different rates of VAT on consumption and transactions services.

5. Calibration

Here, we solve numerically for the steady-state $\tau^c$, $\tau^d$ and other endogenous variables of the model, using calibrated parameters. First, we focus on the steady state, so we can drop time subscripts for all variables. We assume a standard iso-elastic functional form for utility in (3.2) of the form:

$$u(c, l) = \frac{1}{1-\theta}(c^{1-\theta} - 1) + \frac{A}{1-\eta}(l^{1-\eta} - 1), \quad v(g) = \frac{B}{1-\phi}(g^{1-\phi} - 1)$$  \hspace{1cm} (5.1)

The production function is assumed to be Cobb-Douglas, i.e. $f(h, k) = h^\alpha k^{1-\alpha}$. Given these functional forms, and the focus on a steady state, the equilibrium conditions can be written as twelve simultaneous equations in twelve unknowns $(c, l, h, k, \lambda, w, j^*, m, d, \tau^d, \tau^c)$, as described in Appendix A2. The parameters $(A, B, \eta, \theta, \phi, \alpha, \delta, \psi, \sigma, \gamma, r)$ of these equations are calibrated as described in Table 1 below.

We discuss these choices in more detail. First, $\theta$ is the inverse of the elasticity of intertemporal substitution (EIS). In an important and well cited empirical study, Hall (1988) concludes that it is not likely to be larger than 0.1. Other studies use a value of 0.2 (Chari et al., 2002; House and Shapiro, 2006; Piazzesi et al., 2007), or a value of 0.5 (Jin, 2012; Trabandt and Uhlig, 2011; Rudebusch and Swanson, 2012). We take a central value of the EIS of 0.2, giving a value of $\theta$ of 2.5.

Next, $\eta$ is the inverse of the intertemporal elasticity of substitution of leisure (EIL). Empirical studies find the EIL to be less than 1 (Mankiw et al., 1985). We assume a central value of 0.2, giving a mid-value of 2.5 for $\eta$. Next, we set $\phi = 1$. Finally, note that as $\beta = 1/(1 + r)$ in the steady state, $r$ is fixed at 5%.

From the Federal Reserve Economic Data (FRED), the level of currency as percent of GDP is $m = M/Y = 6.1\%$, and deposits as percent of GDP is $d = D/Y = 6.5\%^{21}$. Next, $\psi$ is derived from the following calculation: from the BEA Input-Output tables, the value of financial intermediation services as a percent of GDP is $D\psi/Y = 1.6\%$. Combining these gives $\psi = 0.246$.

---

\(^{21}\) $M$ is calculated as the value of currency from the FRED database, and $D$ as M1 minus currency. All values were for the most recent year for which $Y$ was available, i.e. 2011.
Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Mean Value</th>
<th>Source</th>
</tr>
</thead>
</table>
| $\theta$ | Elasticity of utility w.r.t. consump-
| tion    |                                       | 2.5        | Hall, 1998 and others                       |
| $\eta$   | Elasticity of utility w.r.t. leisure  | 2.5        | Mankiw et al. (1985)                       |
| $\phi$   | Elasticity of utility w.r.t. public
good | 1.0        |                                             |
| $\psi$   | Cost of financial services            | 0.25       | BEA Input-Output tables, US FRED*          |
| $\beta$  | Time discount factor                  | 0.95       |                                             |
| $A$      | Leisure parameter                     | 1.0        | Chosen to calibrate average value for US
economy                                      |
| $B$      | Public good parameter                | 0.5        | as above                                    |
| $\sigma$| parameter of the cost-of-cash function | 5.0        | as above                                    |
| $\gamma$| parameter of the cost-of-cash function | 1.5        | as above                                    |
| $\alpha$| Share of labor**                     | 0.64       | Henriksen and Kydland (2010)                |
| $\delta$| Depreciation**                       | 0.025      | ibid.                                       |

Note: For each parameter, a value was randomly drawn from a uniform distribution with a lower and upper values of 25% around the mean value; * Federal Reserve Economic Data (FRED); ** Parameters that were not randomly sampled.

Finally $A, B, \sigma, \gamma$ were not specified exogenously but chosen so as to match the variables $g, m, d, l, h$ to their respective average values for the US economy. These average values are obtained as follows. First, $m, d$ have already been specified. From the the Office of Management of the Budget, federal government outlays are about 20% of GDP, i.e., $g = G/Y = 20\%$. Next, it is standard to take the fraction of time spent working at around 30%, so we set $h = 0.3$. The Bureau of Labour Statistics Time Use Survey for 2011\(^{22}\) finds that the average time spent shopping across all adults was 0.72 hours per day. As a proportion of the available working day (16 hours), this is approximately 0.05. This implies leisure $l$ is a share $l = 1 - 0.3 - 0.05 = 0.65$ of the total time endowment.

\(^{22}\)http://www.bls.gov/tus/
Finally, we solve this system of equations using a Mixed Complementarity Problem (MCP) algorithm. We begin solving the model using the mean values presented in Table 1, and also perform sensitivity analysis on the various parameters by using a Monte Carlo method. For each parameter, we randomly draw a value from a uniform distribution with lower and upper values using an arbitrarily chosen value of 25% around the mean. The model is re-execute 30,000 times and the results are collected and analyzed. We find that the average value for $\tau_c$ is 50% and for $\hat{\tau}_d$ is 8%. The figure below plots the relative tax ratio (i.e., $\tau_c/\hat{\tau}_d$). Its average is 6.4, with the 95% confidence interval falling between 4.8 to 8.5 and standard deviation of 0.9.

6. Conclusions

This paper has considered the optimal taxation of payment services in a dynamic economy. In our model, the demand for payment services is explicitly modelled via household demand for cash, and for bank deposits. Realistically, both of these media of exchange provide similar, but complementary, transactions services. We assume that cash is costless, but we also assume that the banking sector incurs real resource costs in providing

---

23We used the General Algebraic Modeling System (GAMS), which is a high-level modeling system for mathematical programming and optimization.
deposits and the services associated with them. The question addressed in the paper is thus how to tax the payment services provided by banks.

Our main finding is that transaction services should be taxed at a different rate on consumption goods. Theoretically, this rate could be higher or lower. However, under standard assumptions on parameters from the macroeconomics literature, and calibrating the model to US data, we find that the rate on transactions services should be lower, perhaps only one sixth of the tax on consumption. This finding has implications for the current policy debate on the taxation of banks, especially in Europe, where it is a view of many, including the European Commission, that banks are undertaxed, because many of their services are exempt from VAT. Our results imply that this form of undertaxation may not be of great concern. Of course, there are other reasons for taxing banks, for example, to charge ex ante for the social costs of bailouts, or corrective taxes to discourage excessive risk-taking, and so on.
7. References


Correia, I., and P. Teles: 1996, "Is the Friedman rule optimal when money is an
intermediate good?”, *Journal of Monetary Economics*, 38: 223-244.


A. Appendix

A.1. Proofs of Propositions and Other Results

Proof that $\mu \geq 0$. From (4.14), (4.9), (4.4) we have:

$$u_t(1 + \mu(1 + H_{lt})) = w_t v_{gt}$$

$$\implies \mu = \frac{v_{gt} - u_{lt}/w_t}{1 + H_{lt}}$$

(A.1)

Suppose to the contrary that $\mu < 0$ at the optimum. Then, from (A.1),

$$\frac{1}{1 + H_{lt}} \frac{v_{gt} - u_{lt}/w_t}{u_{lt}/w_t} < 0$$

(A.2)

But, $v_{gt} < u_{lt}/w_t$, utility could be increased if $1\$ of spending on the public good were returned to the household as a lump-sum, contradicting the optimality of the policy. But if $v_{gt} > u_{lt}/w_t$, then the only other possibility is that $H_{lt} < -1$ from (A.2). But by the assumption that $u_{lt} \geq 0, u_{ll} < 0, H_{lt} > 0$ from (4.6). So, this is a contradiction. □

Proof of Proposition 1. Combining (4.10)-(4.12), and using $\beta^t/\xi_t = 1/v_{gt}$ from (4.14) we get

$$\left(w_t - \frac{\mu u_{lt}}{v_{gt}}\right) \phi'(j_t^*) = c_t j_t^* \psi$$

(A.3)

As $\mu > 0$, this implies

$$\psi < \frac{w_t \phi'(j_t^*)}{c_t j_t^*}$$

(A.4)

On the other hand, from (3.10), we have

$$\psi_t^d - \psi_t^m = \frac{w_t(1 - \tau_t^w)\phi'(j_t^*)}{c_t j_t^*}$$

(A.5)

Combining (A.4), (A.5) gives

$$\frac{\psi_t^d - \psi_t^m}{w_t(1 - \tau_t^w)} > \frac{\psi}{w_t}$$

as required.

Proof of Proposition 2. Setting $\psi_t^m = 0$ in (3.10), and using (3.12), we get

$$w_t(1 - \tau_t^w)\phi'(j_t^*) = c_t j_t^* \psi_t^c (1 + z_t^d)$$

(A.6)

Combining (A.3), (A.6), and using $1 + \tau_t^c = 1/(1 - \tau_t^w)$, we get

$$\frac{\tau_t^d}{1 + \tau_t^d} = \frac{\mu u_{lt}}{v_{gt}} - \tau_t^w w_t = (1 + \tau_t^c) \frac{\mu u_{lt}}{w_t v_{gt}} - \tau_t^c$$

(A.7)
Using (A.1) in this last expression, we get
\[ \frac{\tilde{\tau}^d}{1 + \tilde{\tau}^d} (1 + \tau^c) A - \tau^c, \quad A = \left( \frac{v_{gt} - u_{lt}/w_t}{v_{gt}} \right) \]  
(A.8)

But, manipulation of (A.8) gives
\[ \tilde{\tau}^d = (1 + \tau^c)(1 + \tilde{\tau}_d) - 1 = \frac{A}{1 + H_l - A} \]

Then, further rearrangement gives
\[ \frac{\tilde{\tau}^d}{1 + \tilde{\tau}^d} = \frac{A}{1 + H_l} \]
as required. □

Proof of Proposition 3. (i) From (4.4), (4.8)-(4.11), we have:
\[ \frac{W_{ct}}{W_{lt}} = \frac{u_{ct}}{u_{lt}} + \frac{1 + \mu(1 + H_{ct})}{1 + \mu(1 + H_{lt})} = \frac{\xi_t + \xi^d_t K^d_t + \xi^m_t K^m_t}{f_{lt} \xi_t} \]
(A.9)

And, from (3.8),(3.9):
\[ \frac{u_{ct}}{u_{lt}} = \frac{1 + \psi K^d(1 + \tilde{\tau}^d)}{w_t(1 - \tau^w)} \]
(A.10)

So, combining (A.9), (A.10), setting \( 1 + \tau^c_t \equiv 1/(1 - \tau^w_t) \), and recalling that \( \tilde{\tau}^d_t = (1 + \tau^c_t)(1 + \tilde{\tau}_d) - 1 \), we get
\[ (\tau^c_t + \psi K^d_t \tilde{\tau}^d_t)(1 + \mu(1 + H_{ct})) = (1 + \psi K^d_t) \mu(H_{lt} - H_{ct}) \]
(A.11)

Then, adding \((\tau^c_t + \psi K^d_t \tilde{\tau}^d_t)\mu(H_{lt} - H_{ct})\) to both sides, and rearranging, we get:
\[ \frac{\tau^c_t + \psi K^d_t \tilde{\tau}^d_t}{1 + \psi K^d_t + \tau^c_t + \psi K^d_t \tilde{\tau}^d_t} = \frac{\mu(H_{lt} - H_{ct})}{1 + \mu(1 + H_{lt})} \]

Then, using (A.1) to substitute out \( \mu \), and rearranging, we get (4.17) as required. □

Proof of Proposition 6. From (4.8), (3.7), we get
\[ \frac{\beta^{t-1} W_{ct-1}}{\beta W_{ct}} = \frac{1}{\beta B_{lt}} \frac{u_{ct-1}}{u_{ct}} = \frac{\xi_{t-1} + \xi^d_{t-1} K^d_{t-1} + \xi^m_{t-1} K^m_{t-1}}{\xi_t + \xi^d_t K^d_t + \xi^m_t K^m_t} \]
(A.12)

\[ = \frac{\xi_{t-1}(1 + \psi K^d_{t-1})}{\xi_t(1 + \psi K^d_t)} \]
using $\xi_t^d = \psi \xi_t, \xi_t^m = 0$ from (4.10), (4.11) in the second line, and where

$$B_t = \frac{1 + \mu(1 + H_{ct})}{1 + \mu(1 + H_{ct-1})}$$

Moreover, from (4.13) and $f_{kt} - \delta = r_t$, we have:

$$\frac{\xi_{t-1}}{\xi_t} = 1 + f_{kt} - \delta = 1 + r_t \quad (A.13)$$

Combining (A.12) and (A.13), we get:

$$\frac{u_{ct-1}}{u_{ct}} = \beta B_t \frac{(1 + \psi K_{t-1}^d)}{(1 + \psi K_t^d)} (1 + r_t) \quad (A.14)$$

Finally, from (3.8), (??), we get:

$$\frac{u_{ct-1}}{u_{ct}} = \beta (1 + (1 - \tau_t^r) r_t) \frac{1 + K_{t-1}^d \psi (1 + \tilde{\tau}_{t-1}^d)}{1 + K_t^d \psi (1 + \tilde{\tau}_t^d)} \quad (A.15)$$

Combining (A.14), (A.15), and eliminating $\frac{u_{ct-1}}{u_{ct}}$, we get

$$B_t \frac{(1 + \psi K_{t-1}^d)}{(1 + \psi K_t^d)} (1 + r_t) = (1 + (1 - \tau_t^r) r_t) \frac{1 + K_{t-1}^d \psi (1 + \tilde{\tau}_{t-1}^d)}{1 + K_t^d \psi (1 + \tilde{\tau}_t^d)} \quad (A.16)$$

In the steady state, this reduces to $1 + r_t = 1 + (1 - \tau_t^r) r_t$ which of course, implies $\tau_t^r = 0$ as required.

A.2. Calibration Equations

First, we have the market-clearing conditions for the intermediate good and labour;

$$c + \delta k + g + \psi d = h^\alpha k^{1-\alpha} \quad (A.17)$$

$$h = 1 - l - \frac{\gamma (j^*)^\sigma + 1}{\sigma + 1} \quad (A.18)$$

where (A.17) follows directly from (3.13). Second, the firm’s first-order conditions for labour and capital determine the factor prices:

$$w = \alpha \left(\frac{k}{h}\right)^{1-\alpha} \quad (A.19)$$

$$r = -\delta + (1 - \alpha) \left(\frac{h}{k}\right)^\alpha \quad (A.20)$$
To derive the steady-state household budget constraint from (3.5), set all variables independent of \( t \); and set \( m = r = 0 \) from Propositions 1 and 3. This gives:

\[
c + m(1 + \pi) + d(1 + \pi) + k^H = w(1 - \tau^w)h + m + (1 + \tilde{\tau})(1 + \pi)d + (1 + r)k^H
\]

(CA.21)

Cancelling terms in (A.21), we get

\[
c + m\pi = w(1 - \tau^w)h + \tilde{\tau}(1 + \pi)d + rk^H
\]

But, by definition, \( k^H = k - d \), so

\[
c + m\pi = w(1 - \tau^w)h + \tilde{\tau}(1 + \pi)d + r(k - d)
\]

Also, \( \pi = -\frac{\tau}{1 + \tau}, \tilde{\tau} = r - \frac{\psi(1 + r^d)}{1 + \tau} \), so

\[
c - \frac{mr}{1 + r} = \frac{wh}{1 + \tau^c} + (r(1 + \pi) - \psi(1 + \tau^d))d + r(k - d)
\]

Or,

\[
c + (\psi(1 + \tau^d) + r) d = w(1 - \tau^w)h + \frac{r}{1 + r}(m + d) + rk
\]

This is intuitive: the right-hand side says that total household income is income from labour plus income from capital plus a seigniorage subsidy (due to the negative inflation rate). The household first-order conditions (3.8)-(3.10) reduce to:

\[
c^{-\theta} = \lambda(1 + \psi(1 + \tau^d)K^d)
\]

(A.23)

\[
Al^{-\eta} = \lambda w/(1 + \tau^c)
\]

(A.24)

\[
w^2(j^*)^\sigma/(1 + \tau^c) = c^2(j^*\psi(1 + \tau^d))
\]

(A.25)

Next, the transaction constraints (3.3) reduce to

\[
m = c(j^*)^2
\]

(A.26)

\[
d = c (1 - (j^*)^2)
\]

(A.27)

Finally, noting that \( K^d = 1 - (j^*)^2 \), and using (5.1), the optimal tax rules simplify to

\[
\frac{\hat{\tau}^d}{1 + \hat{\tau}^d} = \frac{1}{1 + \eta h/l} \left( \frac{Bg^{-\phi} - Al^{-\eta}/w}{Bg^{-\phi}} \right)
\]

(A.28)

\[
\frac{\tau^c + \psi(1 - (j^*)^2)\hat{\tau}^d}{1 + \tau^c + \psi(1 - (j^*)^2)(1 + \hat{\tau}^d)} = \frac{\eta h/l + \theta}{1 + \eta h/l} \left( \frac{Bg^{-\phi} - Al^{-\eta}/w}{Bg^{-\phi}} \right)
\]

(A.29)

So, we obtain 12 equations (A.17)-(A.20), (A.22)-(A.29) in 12 unknowns:

\( (c, l, h, k, g, \lambda, w, j^*, m, d, \hat{\tau}^d, \tau^c) \), and the parameters \( (A, B, \eta, \theta, \phi, \alpha, \delta, \psi, \sigma, \gamma, r) \).
WP 14/22 Chris Sanchirico As American as Apple Inc: International tax and ownership authority

WP 14/19 Jörg Paetzold and Hannes Taking the High Road? Compliance with commuter tax allowances and the role of evasion spillovers

WP 14/18 David Gamage How should governments promote distributive justice?: A framework for analyzing the optimal choice of tax instruments

WP 14/16 Scott D Dyreng, Jeffrey L Hoopes and Jaron H Wilde Public pressure and corporate tax behaviour

WP 14/15 Eric Zwick and James Mahon Do financial frictions amplify fiscal policy? Evidence from business investment stimulus

WP 14/14 David Weisbach The use of neutralities in international tax policy

WP 14/13 Rita de la Feria Blueprint for reform of VAT rates in Europe

WP 14/12 Miguel Almunia and David Lopez Rodriguez Heterogeneous responses to effective tax enforcement: evidence from Spanish firms

WP 14/11 Charles E McLure, Jack Mintz and George R Zodrow US Supreme Court unanimously chooses substance over form in foreign tax credit

WP 14/10 David Neumark and Helen Simpson Place-based policies

WP 14/09 Johannes Becker and Ronald B Davies A negotiation-based model of tax-induced transfer pricing

WP 14/08 Marko Koethenbuerger and Michael Stimmelmayr Taxing multinationals in the presence of internal capital markets

WP 14/07 Michael Devereux and Rita de la Feria Designing and implementing a destination-based corporate tax

WP 14/05 John W Diamond and George R Zodrow The dynamic economic effects of a US corporate income tax rate reduction

WP 14/04 Claudia Keser, Gerrit Kimpel and Andreas Oesterricher The CCCTB option – an experimental study

WP 14/03 Arjan Lejour The foreign investment effects of tax treaties

WP 14/02 Ralph-C. Bayer Harald Oberhofer and Hannes Winner The occurrence of tax amnesties: theory and evidence
WP14/01 Nils Herger, Steve McCorriston and Christos Kotsogiannisz *Multiple taxes and alternative forms of FDI: evidence from cross-border acquisitions*

WP13/25 Michael Devereux, Niels Johannesen and John Vella *Can taxes tame the banks? Evidence from European bank levies*

WP13/24 Matt Krzepkowski *Debt and tax losses: the effect of tax asymmetries on the cost of capital and capital structure*

WP13/23 Jennifer Blouin, Harry Huizinga, Luc Laeven, Gaëtan Nicodème *Thin capitalization rules and multinational firm capital structure*

WP13/22 Danny Yagan *Capital tax reform and the real economy: the effects of the 2003 dividend tax cut*

WP13/21 Andreas Haufler and Christoph Lülfesmann *Reforming an asymmetric union: on the virtues of dual tier capital taxation*

WP13/20 Michael Blackwell *Do the have come out ahead in tax litigation? An empirical study of the dynamics of tax appeals in the UK*

WP13/19 Johannes Becker and Ronald B Davies *Learning and international policy diffusion: the case of corporate tax policy*

WP13/18 Reuven S Avi–Yonah *And yet it moves: taxation and labour mobility in the 21st century*

WP13/17 Anne Brockmeyer *The investment effect of taxation: evidence from a corporate tax kink*

WP13/16 Dominika Langenmayr and Rebecca Lesterz *Taxation and corporate risk-taking*

WP13/15 Martin Ruf and Alfons J Weichenrieder *CFC legislation, passive assets and the impact of the ECJ’s Cadbury–Schweppes decision*

WP13/14 Annette Alstadsæter and Martin Jacob *The effect of awareness and incentives on tax evasion*

WP13/13 Jarkko Harju and Tuomos Matikka *The elasticity of taxable income and income-shifting between tax bases: what is “real” and what is not?*

WP13/12 Li Liu and Andrew Harper *Temporary increase in annual investment allowance*

WP13/11 Alan J Auderbach and Michael P Devererux *Consumption and cash-flow taxes in an international setting*

WP13/10 Andreas Haufler and Mohammed Mardan *Cross-border loss offset can fuel tax competition*
WP13/09 Ben Lockwood *How should financial intermediation services be taxed?*

WP13/08 Dominika Langenmayr, Andreas Hauffer and Christian J bauer *Should tax policy favour high or low productivity firms?*

WP13/07 Theresa Lohse and Nadine Riedel *Do transfer pricing laws limit international income shifting? Evidence from European multinationals*

WP13/06 Ruud de Mooij and Jost Heckemeyer *Taxation and corporate debt: are banks any different?*

WP13/05 Rita de la Feria *EU VAT rate structure: towards unilateral convergence?*

WP13/04 Johannes Becker and Melaine Steinhoff *Conservative accounting yields excessive risk-taking - a note*

WP13/03 Michael P. Devereux, Clemens Fuest, and Ben Lockwood *The Taxation of Foreign Profits: a Unified View*

WP13/02 Giorgia Maffini *Corporate tax policy under the Labour government 1997–2010*

WP13/01 Christoph Ernst, Katharina Richter and Nadine Riedel *Corporate taxation and the quality of research & development*

WP12/29 Michael P Devereux and Simon Loretz *What do we know about corporate tax competition?*

WP12/28 Rita de la Feria and Richard Krever *Ending VAT Exemptions: Towards A Post-Modern VAT*

WP12/27 Theresa Lohse, Nadine Riedel and Christoph Spengel *The Increasing Importance of Transfer Pricing Regulations – a Worldwide Overview*

WP12/26 Harry Huizinga, Johannes Voget and Wolf Wagner *Capital gains taxation and the cost of capital: evidence from unanticipated cross-border transfers of tax bases*

WP12/25 Harry Huizinga, Johannes Voget and Wolf Wagner *International taxation and cross border banking*

WP12/24 Johannes Becker and Nadine riedel *Multinational Firms Mitigate Tax Competition*

WP12/23 Michael Devereux, Li Liu and Simon Loretz *The Elasticity of Corporate Taxable Income: New Evidence from UK Tax Records*

WP12/21 Peter Egger, Christian Keuschnigg, Valeria Merlo and Georg Wamser Corporate
taxes and internal borrowing within multinational firms

WP12/20 Jarkko Harju and Tuomos Kosonen The impact of tax incentives on the economic
activity of entrepreneurs

WP12/19 Laura Kawano and Joel slemrod The effects of tax rates and tax bases on
corporate tax revenues: estimates with new measures of the corporate tax base

WP12/18 Giacomo Rodano, Nicolas Serrano-Velarde and Emanuele Tarantino Bankruptcy
law and the cost of banking finance

WP12/17 Xavier Boutin, Giacinta Cestone, Chiara Fumagalli, Giovanni Pica and Nicolas
Serrano-Velarde The Deep pocket effect of internal capital markets

WP12/16 Clemens Fuest, Andreas Peichl and Sebastian Siegloch Which workers bear the
burden of corporate taxation and which firms can pass it on? Micro evidence from
Germany

WP12/15 Michael P. Devereux Issues in the Design of Taxes on Corporate Profit

WP12/14 Alan Auerbach and Michael P. Devereux Consumption Taxes In An International
Setting

WP12/13 Wiji Arulampalam, Michael P. Devereux and Federica Liberini Taxes and the
location of targets

WP12/12 Scott Dyreng, Bradley Lindsey and Jacob Thornock Exploring the role Delaware
plays as a tax haven

WP12/11 Katarzyna Bilicka and Clemens Fuest With which countries do tax havens share
information?

WP12/10 Giorgia Maffini Territoriality, Worldwide Principle, and Competitiveness of
Multinationals: A Firm-level Analysis of Tax Burdens

WP12/09 Daniel Shaviro The rising tax-electivity of US residency

WP12/08 Edward D Kleinbard Stateless Income

WP12/07 Vilen Lipatov and Alfons Weichenrieder Optimal income taxation with tax
competition

WP12/06 Kevin S Markle A Comparison of the Tax-motivated
Income Shifting of Multinationals in Territorial and Worldwide Countries

WP12/05 Li Liu Income Taxation and Business Incorporation:
Evidence from the Early Twentieth Century

WP12/04 Shafik Hebous and Vilen Lipatov A Journey from a Corruption Port to a Tax Haven
WP12/03 Neils Johannesen  *Strategic line drawing between debt and equity*

WP12/02 Chongyang Chen, Zhonglan Dai, Douglas A. Shackelford and Harold H. Zhang,  
*Does Financial Constraint Affect Shareholder Taxes and the Cost of Equity Capital?*

WP12/01 Stephen R. Bond and Irem Guceri,  
*Trends in UK BERD after the Introduction of R&D Tax Credits*