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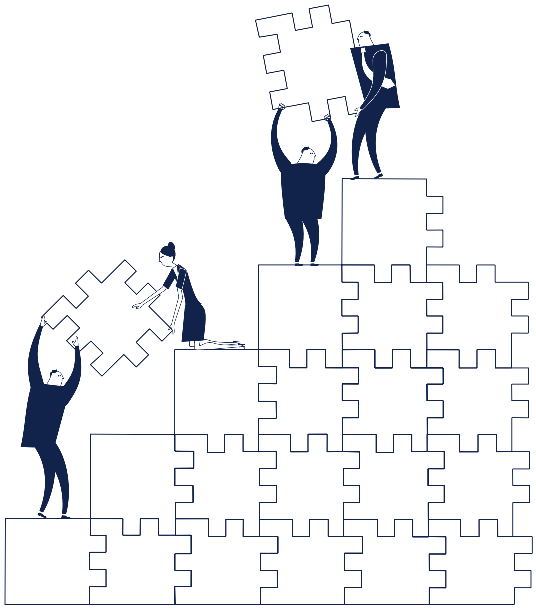

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The Network Composition of Aggregate Unemployment*

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Abstract

We develop an alternative framework to the aggregate matching function in which workers search for jobs through a network of firms: the labor flow network. The lack of an edge between two companies indicates the impossibility of labor flows between them due to high frictions. In equilibrium, firms' hiring behavior correlates through the network, generating highly disaggregated local unemployment. Hence, aggregation depends on the topology of the network in non-trivial ways. This theory provides new micro-foundations for the the Beveridge curve, wage dispersion, and the employer-size premium. Using employer-employee matched records, we find that the empirical topology of the network, in conjunction with the supply elasticity, may be a major contributor of aggregate unemployment.

Keywords: *Labor flows, networks, unemployment, aggregation, job search.*

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1 Introduction

Unemployment is a fundamental economic problem resulting form several distinct social mechanisms. These include people becoming separated from their jobs and searching for new positions; firms opening vacancies and searching for new workers using diverse strategies; and recruiters finding job seekers throughout the labor market. Due to the complexity involved in simultaneously accounting for these and other mechanisms, the composition of unemployment has been studied under the umbrella of labor market frictions. A simplified way to account for these frictions has been to assume that companies and job seekers meet at random in the job market. Failure to coordinate these encounters can then be attributed to frictions.

The seminal work of Hall (1979), Pissarides (1979), and Bowden (1980) paved the way for the application of random matching models in order to integrate frictions into equilibrium models. A reduced way to capture these matching processes is through the aggregate matching function (AMF). In its most typical form, the AMF takes two quantities as inputs: total unemployment and total number of vacancies; and returns the total number of successful matches. If the AMF produces unsuccessful matches, even when there are more vacancies than unemployed, it means that the labor market has frictions. Like any aggregation, the AMF implicitly assumes certain regularity in the matching process. Without such regularity it would be impossible to define such a function since the same number of vacancies could yield significantly different levels of unemployment under the same conditions. Therefore, this reduced representation of the matching process sacrifices structural information about frictions. As we will show in this paper, when the structure of frictions correlates the behavior of firms, important effects on aggregate unemployment can be missed through aggregate approaches.

In order to address the limitations of the AMF, numerous models about its micro-foundations have been formulated. In some cases, they rely on theoretical assumptions that are hard to observe through empirical data. In other cases, micro-foundations can be extremely specific mechanisms that are difficult to extrapolate to more general contexts, or to link to other mechanisms under a common framework (e.g., geographical distance, social networks, skills mismatch, etc.). Furthermore, even when it is possible to account for multiple micro-foundations simultaneously, it does not take much to end up with overly-complicated models. Therefore, developing an overarching framework that accounts for the highly heterogeneous and complex structure of labour market frictions is something desirable from both positive (to understand labor dynamics) and normative (for policy purposes) points of view. In this paper we propose a new framework to achieve this goal, inspired in empirical observations on how individuals move from one company to another throughout their careers.

We propose that the trajectories of individuals across firms provide rich information about the structure of the labor market frictions. We represent such a structure as a network of firms that restricts labor mobility. In this network, the presence or absence of an edge represents a categorical relation between two firms, resulting from the frictions that determine the amount of labor mobility between them. More specifically, the absence of an edge means that labor flows between two unconnected firms are unlikely due to high frictions (at least in the short run), while the opposite is expected for connected firms. Together, firms and edges form the *labor flow network* (LFN) of the economy. In the same spirit in which the AMF uses past employment outcomes to determine today's unemployment, our approach uses past trajectories to determine today's employment outcomes. Another analogy can be drawn with the urns-balls literature. Here the urns are distributed across individual firms, and jumps between them are restricted by the connections between them. In fact, the classical urns-balls model is a special case of ours, where the network topology has a high degree of regularity. Therefore, the topology of the LFN plays a crucial role in determining unemployment. As we will show in this paper, the empirical topology of the LFN, makes aggregation of unemployment non-trivial. Furthermore, when firms' hiring behavior correlate through the LFN, the structure of the labour market frictions may account for large amounts of unemployment.

1.1 Related Works

The idea of limiting job search to groups of firms is not new or uncommon. For example, mismatch models posit that coordination failures between firms and workers are due to frictions that prevent job seekers from freely moving between submarkets. Conventionally, mobility between submarkets is studied by grouping firms into different categories and analyzing the labor flows that take place between such groups. Since the early contribution of Lucas and Prescott (1974), multisector matching models have offered a variety of ways to think about frictions between submarkets. An example can be found in Shimer (2007), where inter-submarket flows are modeled as a process where workers and jobs are randomly reassigned to any submarket every period. This reassignment originates from an exogenous stochastic process under which it is equally likely to move between any two submarkets. Once workers and jobs have been reallocated, matching takes place in each submarket through local AMFs. In contrast, Sahin et al. (2014) assume that, provided with information on vacancies, shocks, and efficiencies, workers periodically choose a submarket to move into. Once labor is reallocated, match creation and destruction take place in each submarket. An alternative approach proposed by Herz and van Rens (2011) assumes that workers can search for vacancies in any submarket and firms can search for workers in the same way. There are costs associated to searching in each submarket. Therefore, matching depends on the optimal decisions of workers and firms about where to search. Other models combine some of these elements in the tradition of Lucas and Prescott (Alvarez and Shimer, 2011; Carrillo-Tudela and Visschers, 2013; Lkhagvasuren, 2009; Kambourov and Manovskii, 2009). On the other hand, a related strand of research studies submarkets as spatially delimited units (generally cities) (Glaeser and Gottlieb, 2009; Moretti, 2011; Manning and Petrongolo, 2011). These models focus on the effect of local shocks when the economy is in spatial equilibrium, which is useful when we know the spatial location of interest. However, as units of aggregation, spatial partitions can be quite arbitrary.

Whether it is for the whole economy or for submarkets, there are a number of problems that arise from viewing matching in aggregate terms, and here we mention a few. First, when an AMF is responsible of pairing up workers and vacancies, it is assumed that all matches are equally likely. This neglects the importance that specific firms have in reallocating labor within a submarket. Second, defining a submarket is an arbitrary choice that might be well suited for a specific problem, but not necessarily for a broader context. Since these classifications are usually build for taxonomic purposes, they are not designed to minimize inter-submarket flows and maximize intra-submarket, which would capture the structural information of labour market frictions. This problem has been pointed out by Jackman and Roper (1987) in their classical paper on structural unemployment:

> ... "there seems no particular reason why unemployed workers should regard themselves as specific to a particular industry, and in practice the unemployed do move between industries reasonably easily." (Jackman and Roper, 1987, pg. 19)

Third, aggregation assumes that any worker from one submarket is equally likely to transition to another submarket. Furthermore, it ignores the fact that only a few firms are responsible for inter-submarket transitions. These firms are crucial to overall labor mobility since they are diffusion outlets or bottlenecks in the process of labor reallocation. Fourth, aggregation 'smooths' the search landscape, enabling firm-to-firm flows that are highly unlikely in the short run. In fact, Guerrero and López (2015) have shown that the hypothesis of an AMF is rejected as an explanation of empirical firm-to-firm flows, even at the level of submarkets. Using community detection methods for network data, independent studies by Guerrero and Axtell (2013) and Schmutte (2014) show that conventional classifications such as industries and geographical regions poorly capture the clusters of labor that are detected in employer-employee matched micro-data. For these reasons, a framework that does not rely on arbitrary aggregations to define submarkets would represent a significant methodological improvement. Petrongolo and Pissarides (2001) suggests the use of graph theory as a potential tool to overcome arbitrary aggregations. We take this approach in order to depart from the established notions of submarkets and instead look at labor dynamics as random walks on graphs.

1.2 A Network Approach

Our approach is inspired in a local job search mechanism. When a person looks for a job in search of a vacancy, he or she approaches a group of firms that are 'accessible' in the short run. Such group is determined by the frictions of the labor market and we assume that it is specific to the firm where this person was last employed. We represent the correspondence between firms and their respective groups of accessible firms through a LFN. In this network, firms are represented by nodes. An edge between firms i and j means that frictions are such that j will be accessible to employees of i and vice versa. Therefore, edges have a categorical nature that represents the possibility (or impossibility in their absence) of labor flows between firms. Firm i's edges determine its first neighbors, which are equivalent to the group of accessible firms to someone employed in i. We refer to these firms as i's neighbor firms. As a person progresses through his or her career, he or she traverses the economy by taking jobs at the neighbor firms of past employers. This gradual navigation process is fundamentally different from previous approaches because the identity of the firm (i.e., its position in the LFN) matters in order to determine the employment prospects of the job seeker. There is a number reasons why this is important. To mention a few, it allows to study the composition of aggregate unemployment at the level of the firm; it sheds light on the effect of localized shocks and targeted policies; and it exploits the granularity and inter-firm structure captured in employer-employee matched records. By analyzing the steady-state equilibrium, we obtain analytical solutions that inform us about local unemployment, local flows, firm sizes, profits, and firm hiring behavior. In addition, this framework provides new micro foundations of stylized facts such as the Beveridge curve and the employer-size premium.

Network theory has been extensively used to study labor markets in the context of information transmission through social networks. The pioneering work of Granovetter

(1973) showed the importance that infrequently-used personal contacts have in acquiring non-redundant information about vacancies. Although Granovetter's hypothesis has been recently challenged by studies that use comprehensive social media micro-data (Gee et al., 2014), the importance of social networks in diffusing job information is not in question. Other empirical studies about social networks in labor markets look at migration (Munshi, 2003), urban and rural unemployment (Wahba and Zenou, 2005), investment in personal contacts (Galeotti and Merlino, 2014), and local earnings (Schmutte, 2010) among other topics. On the theoretical side, there is a substantial number of models concerning social networks in labor markets, pioneered by Boorman (1975) and Montgomery (1991b). Some studies have focused on labor outcomes as a result of the structure of social networks (Calvó-Armengol and Jackson, 2004; Calvó-Armengol and Zenou, 2005; Calvó-Armengol and Jackson, 2007; Schmutte, 2010). Other works analyze inequality and segregation effects in the job market (Calvó-Armengol and Jackson, 2004; Tassier and Menczer, 2008). For a review of these and other models, we refer to the literature survey provided by Ioannides and Loury (2004).

Despite the wide application of network methods to study labor markets, most of this work was only focused on the role of social networks in communicating information about vacancies. These studies have important applications in long-term policies such as affirmative action laws, but are not so useful for short-term policies such as contingency plans in the presence of shocks. Furthermore, the role of the firm in these models becomes trivial if not absent, which is problematic for policies that aim at incentivizing firms. In fact, little has been done to study labor mobility on networks. To the best of our knowledge, there are only a few studies that analyze labor flows through networks. For example, Guerrero and Axtell (2013) study firm-to-firm labor flows using employee-employer matched records from Finland and Mexico. They characterize the topology of these labor flow networks and find that network connectivity is highly correlated with employment growth at the firm level. Using US micro-data, Schmutte (2014) constructs job-to-job networks in order to identify four job clusters. Mobility between these clusters is highly frictional and dependent on the business cycle. Both studies find that any clusters identified through community detection methods have little correspondence to standard categorizations such as industrial classification, geographical regions, or occupational groups. The LFN framework provides a new way to analyze labor dynamics, while contributing to the use of methods from network science in economics.

Our work complements five strands of literature. First it adds to the family of search and matching models in labor economics by introducing the method or random walks on graphs as a new tool to analyze labor mobility and aggregate unemployment. It also pushes the boundaries on how employer-employee matched micro-datasets are used today. Second, it contributes to the field of networks in labor markets by expanding the application of network methods beyond the scope of personal contacts. Social networks are difficult to observe at a large-scales[1]. Since LFNs partially capture labor flows induced by personal contacts (people who worked together may recommend each other in the future), they serve as an additional source of information to study the effect of social networks in the labor market. Third, it complements the literature on micro-foundations of the AMF (Butters, 1977; Hall, 1979; Pissarides, 1979; Montgomery, 1991a; Lang, 1991; Blanchard and Diamond, 1994; Coles, 1994; Coles and Smith, 1998; Stevens, 2007; Naidu, 2007). Because frictions are captured in the form of a network, there is no need to assume an aggregate matching process. Fourth, it strengthens the growing literature of inter-firm networks (Saito et al., 2007; Konno, 2009; Atalay et al., 2011; Acemoglu et al., 2012; di Giovanni et al., 2014). By avoiding aggregation into arbitrary submarkets, the network approach allows to study firm and labor dynamics jointly. Fifth, it contributes to the study of local labor markets by providing a new way of defining localities at the level of the firm, which should facilitate the study of local shocks and their propagation.

This paper is organized in the following way. Section 2 presents the model in two parts. First, we introduce the problem of the firm, which maximizes profits in the steady-state when wages are exogenous. Then, we characterize the job search process as a random walk

[1]Although online social networks provide a rich source of information, they are highly susceptible to biases and multiple factors that incentivize individuals to opt out of this form of communication.

on a graph, which helps us to solve the firm's problem of choosing the optimal number of vacancies.

In section 3, we use employer-employee matched micro-data to test the model's predictions. We find that our results are significant and robust across 20 annual cross-sections of data. In section 4 we endogenize wages and find that firms' hiring behavior correlates through the LFN. Furthermore, when this correlation happens on a topology with a skewed degree distribution, strong effects on aggregate unemployment take place. We construct an empirical LFN and find that its structure is responsible for a significant part of aggregate unemployment in the presence of an inelastic labor supply. Finally, in section 5 we discuss the results, their policy implications, and potential of this framework for future research.

2 Model with an Exogenous Wage

The model considers an economy in a steady state where firms demand a constant amount of labour and workers search for jobs randomly. Since we are interested in understanding the effect of the network topology on unemployment, we focus on firm behavior and model workers as random walkers. In this section, we assume a single exogenous wage, and in section 4 we introduce an exogenous supply to generate wage dispersion.

2.1 Firms

There are $\{1, ..., N\}$ firms in the economy. In the steady state, each firm has size L_i. Every period, a fraction of the i's employees becomes separated with an exogenous probability λ. By hiring job applicants, the firm compensate the loss of employees. Profits are made exclusively from labor rents. Therefore, firms maximize profits by determining the optimal number of vacancies to open every period. However, opening vacancies also depends on exogenous shocks in the form of investments. These investments enable firms to open vacancies and they arrive with a probability v. Therefore, when a firm has vacancies we say that it is *open*, and *closed* otherwise. The expected steady-state firm size is

$$(1-\lambda)L_i + vh_iA_i,$$

where A_i is the number of job applicants and h_i is the fraction of applicants hired by the firm.

Unfilled vacancies are destroyed every period, so we use h_i is a continuous approximation for vacancies in a firm. The intuition is that the firm has an expectation about the number of applicants that it would receive in the steady state; for example, by counting the CVs that job seekers drop at its offices everyday. Hence A_i is the expected number of applicants. The firm opens at most A_i positions in order to minimize the cost of unfilled vacancies (of course there still can be unfilled vacancies if the number of applicants is lower than expected). Therefore, the number of vacancies opened by the firm can be written as a fraction of A_i. We assume independence between workers, so we can treat h_i as a probability. We call it the *hiring policy*, and it represents the likelihood that a job seeker who applies to firm i becomes hired.

Building on a model developed by (Barron et al., 1987), we assume that the objective of the firm is to maximize profits by setting an optimal hiring policy. For this, the firm also takes into account its linear technology with a productivity factor y, the exogenous wage w, and cost parameters $c \in (0,1)$ and $\kappa \in [0,1]$. Then, the profit maximization problem is given by

$$\max_{h_i} \Pi_i = (1-\lambda)(y-w)L_i + v(y-w)h_iA_i - vcL_ih_i - (1-v)\kappa cL_ih_i. \qquad (1)$$

On one hand, c captures the cost of opening more vacancies. We assume that this cost scales with firm size (e.g., because larger firms invest more in screening processes and HR in general), as suggested by recent empirical evidence (Muehlemann and Pfeifer, 2016). On the other hand, κ represents a sunk cost from HR in which the firm incurs when it is closed (e.g., setup expenses for screening future applications).

In order generate concavity in eq. (1), we assume that the firm understands the job search process. Therefore, we proceed to characterize job search and obtain the steady-state solutions for L_i and A_i, which the firm takes into account in order to maximize its profits.

2.2 Job Search

Let us consider a network where each node represents a firm and the absence of an edge between two firms means that labor market frictions between them are so high that we would not expect any labour flows between them in the short run. This network is represented by the adjacency matrix $\mathbf{A}$, where $\mathbf{A}_{ij} = \mathbf{A}_{ji} = 1$ if firms i and j share and edge, and $\mathbf{A}_{ij} = \mathbf{A}_{ji} = 0$ otherwise. Workers flow through this network as they gain and lose employment from its nodes, hence the name of labor flow network (LFN). The LFN is unweighted because edges represent a categorical aspect of the labor market: whether we should expect labor flows between two firms or not. It is undirected because the edges capture some 'affinity' between firms such that frictions are low in both directions. For simplicity, we do not allow self-loops, so the diagonal entries of $\mathbf{A}$ are all zero. We assume that the LFN has a single component. However, the results are generalizable for networks with multiple components. Firm i has $k_i = \sum_j \mathbf{A}_{ij}$, also known as the degree of i. The set Γ_i contains all firms j such that $\mathbf{A}_{ij} = 1$.

Workers can be in one of two states: *employed* or *unemployed*. Regardless of his or her state, each worker is always associated with a firm. Therefore, jobless workers are associated to their last employers. Each worker employed by firm i might become unemployed with probability λ. If unemployed, he or she looks at the set $\gamma_i \subseteq \Gamma_i$ of i's neighbor firms that received investments. Hence, we say that γ_i is the set of open neighbors of i and it may change from one period to another. If $|\gamma_i| = 0$, the job seeker remains unemployed for the rest of the period. Otherwise, he or she selects a firm $j \in \gamma_i$ at random with uniform probability and submits a job application. For simplicity, we assume that each job seeker can submit at most one application per period. It is possible to return to i as long as the last job was held at j such that $\mathbf{A}_{ij}$. Finally, if the job application is successful (with probability h_j), the job seeker becomes employed at j, updating its firm association. Otherwise, it remains unemployed for the rest of the period. We summarize this process in the following steps.

1. Each firm receives an investment with probability v.
2. Each employed worker becomes unemployed with probability λ.
3. Each unemployed associated to i (excluding the newly separated ones) picks a firm $j \in \gamma_i$ at random and becomes employed with probability h_j.

The reader may be concerned about the possibility that a job seeker may occasionally search among firms that are not connected to his or her last employer. If the probability of such event is low, the model preserves the roughly the same characteristics because the LFN induces a dominant effect on job search. When this probability is large, the model becomes an 'urns-balls' model, so the structure of the network is irrelevant. What should be the empirically relevant magnitude of such probability? Previous work shows that the idea of searching on a network is empirically compelling since firm-to-firm labor flows tend to be significantly persistent through time (López et al., 2015). In fact, unrestricted random matching between firms and workers is formally rejected when looking at employer-employee matched records (Guerrero and López, 2015). These results suggests that, in a more general model, the probability of searching 'outside' of the network has to be calibrated with a low value. Such a model can be easily constructed, but its solutions do not have explicit form. In contrast, focusing exclusively on job search 'on' the network yields explicit solutions, which is convenient for building economic intuition.

2.3 Dynamics

The stochastic process previously described is a random walk on a graph with waiting times determined by the investment shocks v, the separation rate λ, and the set of hiring policies $\{h_i\}_{i=1}^N$. In order to characterize its dynamics, we concentrate on the evolution of the probability $p_i(t)$ that a worker is employed at firm i in period t, and the probability $q_i(t)$ that a worker is unemployed in period t and associated to firm i. For this purpose, let us first construct the dynamic equations of both probabilities to then obtain the steady-state solution.

In period t, the probability that a worker is employed at firm i depends on the probability $(1-\lambda)p_i(t-1)$ that he or she was employed at the same firm in the previous period and did not become separated. In case that the worker was unemployed during $t-1$, then $p_i(t)$ also depends on: the probability $q_i(t-1)$ that the worker was associated to a neighbor firm j; on the probability $\Pr(\gamma_j^{(i)})$ of having a particular configuration $\gamma_j^{(i)}$ of open and closed neighbors of j such that i is open; and on the probability $1/|\gamma_i^{(i)}|$ that the worker picks i from all of j's open neighbors. Altogether, summing over all possible neighbors and all possible configurations of open neighbors, and conditioning to the hiring policy, the probability that a worker is employed by firm i in period t is

$$p_i(t) = (1-\lambda)p_i(t-1) + h_i \sum_{j\in\Gamma_i} q_j(t-1) \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) \frac{1}{\left|\gamma_j^{(i)}\right|}, \tag{2}$$

where $\{\gamma_j^{(i)}\}$ denotes the set of all possible configurations of open and closed neighbors of j where i is open.

The probability that a worker is unemployed during t while associated to firm i depends on the probability $\lambda p_i(t-1)$ of becoming separated from i in the previous period. On the other hand, if the worker was already unemployed, the probability of remaining in such state depends on: the probability $\Pr(\gamma_i = \emptyset)$ that no neighbor firm of i is open and the probability $1-h_j$ of not being hired by the chosen open neighbor j. Accounting for all possible non-empty sets γ_i of open neighbors, the probability of being unemployed in t and associated to firm i is given by

$$q_i(t) = \lambda p_i(t-1) + q_i(t-1)\left[\sum_{\gamma_i \neq \emptyset} \Pr(\gamma_i)\frac{1}{|\gamma_i|}\sum_{j\in\gamma_i}(1-h_j) + \Pr(\gamma_i = \emptyset)\right]. \tag{3}$$

Up to this point, the model might seem complicated due to all the parameters involved. However, our intention is to provide a general framework that allows the user to control for different degrees of freedom. As we will show ahead, the steady-state solutions take very simple and intuitive forms, while several parameters can be disregarded if no data is available to measure them. Generally speaking, the qualitative nature of our results holds for different calibrations.

2.4 Steady State

In the steady-state, $p_i(t) = p_i(t-1) = p_i$ and $q_i(t) = q_i(t-1) = q_i$ for every firm i. The following results follow from solving eqs. (2) and (3).

Proposition 1. *The process specified in section 2.2 has a unique steady-state where probabilities p_i and q_i are time-invariant for every firm i.*

Existence follows from a standard result in random walks on graphs (Bollobás, 1998) (see appendix). Uniqueness comes from condition

$$1 = \sum_{i=1}^{N} p_i + \sum_{i=1}^{N} q_i,$$

which indicates that all probabilities should add up to one, implying that every worker is either employed or unemployed, and associated to only one firm. This result implies that a unique steady-state is always reached regardless of how the hiring policies in $\{h_i\}_{i=1}^{N}$ are assigned to each firm in the LFN. López et al. (2015) provide more general results for heterogeneous separation rates and heterogeneous investment shocks. However, this version is more suitable for economic modeling because it yields explicit solutions with intuitive economic meaning.

Proposition 2. *The steady-state average size of a firm i that follows eqs.* (2) *and* (3) *is*

$$L_i = \frac{\varphi}{\lambda} h_i \bar{h}_{\Gamma_i} k_i, \tag{4}$$

where $\bar{h}_{\Gamma_i} = \frac{1}{k_i} \sum_j \mathbf{A}_{ij} h_j$ is the average hiring policy of i's neighbor firms and φ is a normalizing constant.

For now, let us defer the explanation of φ for a few paragraphs. Equation (4) suggests that, *ceteris paribus*, the size of a firm increases with its degree. As expected, firms can increase their own sizes through larger hiring policies. Equation (4) captures an externality: a firm's hiring policy affects the size of its neighbor firms. This result follows from an intuitive mechanism. If firm i hires more people from its pool of applicants, it increases its own size. In consequence, more people will become separated from i through the exogenous separation process governed by λ (which also reduces the size of the firm). More unemployed individuals associated to i translates into a larger pool of job seekers that will potentially apply for a job at i's neighbor j. Therefore, if everything else is constant, A_j increases, contributing to j's growth. This mechanism becomes evident in the following result.

Proposition 3. *The steady-state average number of applications received by a firm i that follows eqs.* (2) *and* (3) *is*

$$A_i = \varphi \bar{h}_{\Gamma_i} k_i. \tag{5}$$

The proof follows from the fact that, in the steady-state, the number of separated employees λL_i must equal the number of newly hired ones $h_i A_i$ in order for L_i to remain constant through time (see appendix).

2.5 Hiring Policy and Profits

We assume that firms understand the job search process to a fair extent. That is, they use eqs. (4) and (5) in eq. (1) and take φ and $\bar{h}_{\Gamma_i}$ as given. Then, substituting eqs. (4) and (5) in eq. (1), and solving the F.O.C. yields the optimal hiring policy

$$h^* = \frac{\psi}{2\phi}(y - w), \tag{6}$$

where $\psi = (1 - \lambda + v\lambda)$ and $\phi = c(v + \kappa - v\kappa)$. We have removed sub-index i because the optimal hiring policy is independent of k_i. This result is quite intuitive in a neoclassical sense, since higher wages are compensated with lower hiring policies. It also suggests that, with a unique exogenous wage, all firms set the same optimal hiring policy. This means that we can rewrite some of these results exclusively as functions of k_i. More specifically, we rewrite the firm size as

$$L_i = \varphi h^{*2} k_i, \tag{7}$$

and the profit as

$$\Pi_i^* = \frac{\varphi\psi^3}{8\lambda\phi^2}(y - w)^3 k_i, \tag{8}$$

which later will be useful for empirical testing.

2.6 Aggregation of Unemployment

Solving eqs. (2) and (3) yields the average number of unemployed individuals associated to firm i in the steady-state. This is a bottom-up construction of unemployment that takes into account how it is distributed across firms, so we term it *firm-specific unemployment*. This new measure provides information about the employment prospects of a firms' ex-employees and a method to identify pools of local unemployment. Firm-specific unemployment is obtained from the following result.

Proposition 4. *The steady-state average unemployment associated to a firm i that follows eqs.* (2) *and* (3) *is*

$$U_i = \frac{\varphi h_i k_i}{1 - (1 - v)^{k_i}}. \tag{9}$$

The normalizing constant φ captures the population conservation condition $H = \sum_i L_i + \sum_i U_i$, so it takes the form

$$\varphi = \frac{H}{\sum_i h_i \bar{h}_{\Gamma_i} k_i \left[\frac{1}{\lambda} + \frac{1}{\bar{h}_{\Gamma_i}[1-(1-v)^{k_i}]}\right]}. \tag{10}$$

Equation (9) becomes more intuitive when multiplying by $\frac{\lambda \bar{h}_{\Gamma_i}}{\lambda \bar{h}_{\Gamma_i}}$, in which case we obtain

$$U_i = \frac{\lambda L_i}{\bar{h}_{\Gamma_i}[1 - (1 - v)^{k_i}]}. \tag{11}$$

Note that $\bar{h}_{\Gamma_i}[1 - (1 - v)^{k_i}]$ is the transition probability from unemployment to employment for a worker associated to firm i. The reciprocal of this probability is the average duration $\bar{t}_i^u$ of an unemployment spell for an individual whose last job was in i. Therefore, we can rewrite eq. (9) as

$$U_i = \lambda \bar{t}_i^u L_i, \tag{12}$$

which will become useful for empirical testing. In general, firm-specific unemployment is an interesting measure because it not only provides a highly granular unit of the composition of aggregate unemployment, but also yields information about how good will be the employment prospects of someone working at a particular company.

Due to the independence between degree and hiring policy implied by eq. (6), aggregation of unemployment is straightforward given that the firm-specific unemployment rate is defined as

$$u_i = \frac{U_i}{U_i + L_i} = \frac{\lambda}{\lambda + h^*[1 - (1 - v)^{k_i}]}, \tag{13}$$

which is non-increasing and convex in k_i. Note that for a LFN where all firms have the same degree, eq. (13) is equivalent to Beveridge curve obtained in 'urn-balls' models.

Let the LFNs of two economies be represented by their adjacency matrices $\mathbf{A}$ and $\mathbf{A}'$, with corresponding degree distributions P and P', and aggregate unemployment rates $u = \sum_{k=1}^{k_{\max}} u_k P(k)$ and $u' = \sum_{k=1}^{k_{\max}} u_k P'(k)$. Then, the next results follow from network stochastic dominance (Jackson and Rogers, 2007a,b; López-Pintado, 2008).

Proposition 5. *If P strictly first-order stochastically dominates P', then $u < u'$.*

Proposition 5 is quite intuitive since the average firm connectivity of $\mathbf{A}$ is higher than in $\mathbf{A}'$. An LFN with higher connectivity reflects an economy with lower labor market frictions. Under these conditions, job seekers have better chances of finding open firms and new job opportunities.

Proposition 6. *If P' is a strict mean-preserving spread of P, then $u < u'$.*

Proofs of propositions 5 and 6 follow from direct differentiation of eq. (13), which shows that u is non-increasing and convex in k_i. Proposition 6 means that more degree heterogeneity translates into higher unemployment. Heterogeneity in a LFN reflects the 'roughness' of the search landscape. It is analogous to heterogeneity in search and matching models. However, there is the fundamental difference: agents traverse the economy by gradually navigating the LFN, instead of being randomly allocated to any firm. As we will learn ahead, this subtle difference in the reallocation process induces significant effects in aggregate unemployment when hiring policies are heterogeneous. We will show that the LFN not only has an ordinal effect on aggregate unemployment, but also a significant impact on its overall level.

3 Empirical Support

In this section, we test the model's predictions when wages are considered exogenous and homogeneous. For this purpose, we use employer-employee matched micro-data from two countries. Given the simple form of our results, these tests should not be interpreted as an attempt to provide definite empirical measures. Instead, we use them as a way to show that our theory is empirically sound (a "sanity check").

3.1 Data

We use different datasets of employer-employee matched records. The first is the Finnish Longitudinal Employer-Employee Data (FLEED), which consists of an annual panel of employer-employee matched records of the universe of firms and employees in Finland. The panel was constructed by Statistics Finland from social security registries by recording the association between each worker and each firm (enterprise codes, not establishments), at the end of each calendar year. If a worker is not employed, it is not part of the corresponding cross-section. The result is a panel of 20 years that tracks every firm and every employed individual at the end of each year (approximately 2×10^5 firms and 2×10^6 workers).

FLEED can be merged with other datasets that provide information about companies. For this, we employ Statistics Finland's Business Register, an annual panel providing the average number of employees and net profits per firm. The Business Register is constructed from administrative data from the Tax Administration, and from direct inquiries from Statistics Finland to business with more than 20 employees. FLEED and the Business Register provide data on labor flows, firm sizes, and net profits from different sources. Unfortunately, their temporal aggregation prevents us from measuring firm-specific unemployment because it is not possible to observe whether a person underwent an unemployment spell between jobs. For this purpose, we employ an additional dataset.

We use a dataset from Mexico consisting of employer-employee matched records with daily resolution. The data was obtained by sampling raw social security records from the Mexican Social Security Institute. Approximately 4×10^5 individuals who were active between 1989 and 2008 were randomly selected and their entire employment history was extracted (hence, covering dates prior to 1989). This procedure generates a dataset with nearly 2×10^5 firms. The records contain information about the exact date in which a person became hired/separated by/from a firm. Therefore, it is possible to identify unemployment spells, duration of each spell, and associations between job seekers and their last employer.

3.2 Empirical Testing

In previous studies we have constructed LFNs by performing different statistical tests about the significance of flows between firms Guerrero and López (2015); López et al. (2015). Since the topology of the network is quite robust under different tests, here we take a simpler approach. For a given year, we construct an edge between two firms if we observe labor flows between them. Therefore k_i is the total number of firms to(from) which firm i sent(received) workers in a given year. Then, we test if indeed there is a positive correlation between the degree of a firm and: its size, its number of unemployed, and its profits. We are aware of potential problems in testing the relationship between degree and firm size (or unemployed), since both are measure from similar data. Furthermore, by testing the relationship between profits and degree we provide a cleaner confirmation of the empirically soundness of our approach.

We test the prediction concerning degree and firm size, expressed in eq. (7) by estimating the model

$$L_i = \beta_L k_i + \epsilon_i, \tag{14}$$

where ϵ_i is an error term and $\beta_L = \varphi h^{*2}$. If the model is empirically consistent, then the null hypothesis of $\beta_L = 0$ should be rejected. Firm sizes are measured as the average number of employees that each firm had in a given year. In a similar spirit, we test the predicted positive relationship between degree and profits suggested in eq. (8) by estimating the model

$$\Pi_i = \beta_\Pi k_i + \epsilon_i, \tag{15}$$

where $\beta_\Pi = \frac{\varphi\psi^3}{8\lambda\phi^2}(y-w)^3$ and profits Π_i are measured in Euros.

We estimate both models for each annual cross-sections in FLEED. Table 1 shows that the data validates the model's prediction eq. (7) in all cross-sections, confirming a positive relationship between degree and firm size. Equation (8) is valid in most of the cross-sections. The fact that profits and degrees are measures of different nature (as opposed to sizes and degrees) gives us provide some level of validity to our approach.

We proceed to test the theoretical prediction connecting degree and firm-specific unemployment, as expressed in eq. (9). For this, we estimate the model

$$U_i = \beta_\lambda x_i + \epsilon_i, \tag{16}$$

where $\beta_\lambda = \lambda$ is the estimated separation rate and $x_i = \bar{t}_i^u L_i$. Like in eq. (14), firm size is measured as the average number of employees associated to a firm in a given year.

Using data from Mexico, we measure the number of associated employed and unemployed individuals U_i in each firm during a single day of every year. By measuring for a single day, we guarantee that the unemployed individuals are different from the employed ones. For

Year	Firm Size β_L	N	R^2	Firm Profit β_Π	N	R^2
1988	7.135*** (4.963e-01)	19,251	0.466	9.031e+04** (4.963e-01)	19,251	0.060
1989	8.200*** (6.766e-01)	19,184	0.549	1.352e+05*** (6.766e-01)	19,184	0.136
1990	11.595*** (1.294e+00)	16,164	0.398	2.291e+05*** (1.294e+00)	16,164	0.088
1991	15.706*** (1.409e+00)	12,752	0.437	2.975e+05*** (1.409e+00)	12,752	0.059
1992	17.776*** (1.620e+00)	12,306	0.416	2.250e+05 (1.620e+00)	12,306	0.019
1993	15.763*** (1.746e+00)	13,750	0.248	2.662e+05* (1.746e+00)	13,750	0.030
1994	15.156*** (2.320e+00)	16,074	0.461	3.074e+05*** (2.320e+00)	16,074	0.049
1995	15.308*** (2.278e+00)	16,675	0.432	2.561e+05*** (2.278e+00)	16,675	0.049
1996	14.193*** (1.879e+00)	20,188	0.541	3.710e+05*** (1.879e+00)	20,188	0.097
1997	11.478*** (1.436e+00)	20,220	0.627	2.405e+05* (1.436e+00)	20,220	0.065
1998	10.803*** (1.902e+00)	25,485	0.570	4.247e+05* (1.902e+00)	25,485	0.069
1999	8.509*** (1.147e+00)	27,340	0.380	7.583e+05* (1.147e+00)	27,340	0.076
2000	9.074*** (1.577e+00)	27,575	0.419	2.506e+05** (1.577e+00)	27,575	0.024
2001	10.879*** (2.048e+00)	26,882	0.500	7.449e+04 (2.048e+00)	26,882	0.007
2002	10.229*** (1.669e+00)	26,546	0.515	5.255e+05 (1.669e+00)	26,546	0.042
2003	10.021*** (1.661e+00)	27,350	0.518	3.511e+05 (1.661e+00)	27,350	0.036
2004	8.855*** (1.751e+00)	29,719	0.506	2.245e+05 (1.751e+00)	29,719	0.021
2005	7.991*** (1.180e+00)	34,089	0.596	2.172e+05 (1.180e+00)	34,089	0.032
2006	7.221*** (9.297e-01)	36,813	0.575	3.474e+05* (9.297e-01)	36,813	0.021

Table 1: Empirical test of theoretical predictions eqs. (7) and (8). The corresponding estimated models are eqs. (14) and (15). Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

each firm, we choose the day when they have the maximum amount of both employed and unemployed, in other words the day of the year that maximizes $U_i L_i$ for each firm i. We compute $\bar{t}_i^u$ by averaging the duration (in number of days) of unemployment spells (shorter than 24 months) associated to firm i. If the sample size of unemployment spells per firm is high, the total number of firms in the sample becomes too low. On the other hand, the data becomes highly noisy (many firms with $U_i L_i \leq 1$) if the sample size of unemployment spells per firm is too low. Therefore, we select firms with at least 80 associated unemployment spells in order to maximize both the number of unemployment spells per firm and the number of firms in the sample.

Table 2 shows that the theoretical prediction in eq. (12) is empirically consistent. Moreover, all the estimated separation rates fall in the interval $(0, 1)$, which is reassuring if we think of the model as a new way to estimate the separation rate.

Figure 1 shows scatter plots of the micro-data for a representative year against our theoretical predictions. It is clear that the predicted relationships are not only statistically significant but positive. Each panel corresponds to an annual cross-section of the datasets: panel A corresponds to eq. (7), panel B to eq. (8), and panel C to eq. (12).

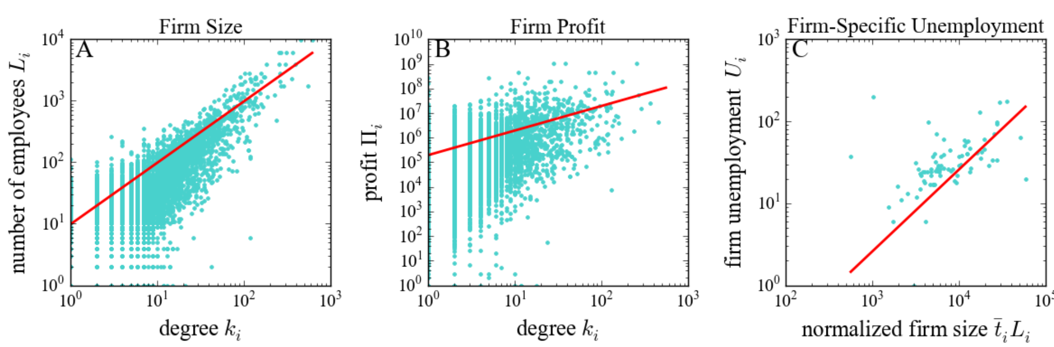
4 Endogenous Wages

So far we have considered a unique exogenous market wage. This allows us to gain new insights about the effect of the LFN on firm dynamics and unemployment. One of the main findings is an externality through which firms affect their neighbors' sizes through their hiring policies. However, it is important to study how this externality interacts with the LFN

Year	β_λ	N	R^2
1999	0.001* (4.203e-04)	83	0.457
2000	0.001*** (3.726e-04)	89	0.502
2001	0.002*** (3.153e-04)	92	0.506
2002	0.002*** (3.448e-04)	93	0.581
2003	0.002*** (3.250e-04)	93	0.729
2004	0.002*** (3.894e-04)	84	0.731
2005	0.003*** (4.936e-04)	86	0.610
2006	0.003*** (6.643e-04)	80	0.507
2007	0.003*** (4.880e-04)	76	0.695
2008	0.003*** (5.379e-04)	72	0.702
2009	0.001** (3.868e-04)	69	0.426
2010	0.001*** (2.242e-04)	63	0.670

Table 2: Empirical test of theoretical prediction eq. (12). The corresponding estimated model is eq. (16). Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Figure 1: Model Prediction vs. Empirical Data



The figures were constructed using the cross-section corresponding to 2006 of each dataset. The scattered dots correspond to empirical observations and the solid lines to the fitted models eqs. (14) to (16).

topology when h_i^* is heterogeneous. For this purpose, we endogenize wages by introducing an exogenous labour supply that determines the wages depending of the amount of labor demanded by each firm. Equilibrium wages are formed when the individual firm demands meet the labour supply, generating a dispersion that depends on the network topology.

For simplicity, we adopt a labor supply with a functional form that guarantees a wage bounded by $(0, 1)$. However, any other monotonically increasing function can be used, as long as the necessary considerations are made in order to guarantee wages and hiring policies with consistent bounds. The inverse labor supply has the form

$$w_i = \frac{a\ell_i}{b + \ell_i}, \tag{17}$$

where ℓ_i is the individual demand of firm i; $b > 0$ is a parameter that affects the price elasticity; and a provides the upper bound of the wage. We assume $a = y$ for analytical convenience, guaranteeing non-negative rents from labor.

The labor demand of firm i is equivalent to the number of new hires. Firms are wage takers, so their profit-maximization problem remains unchanged. Therefore the labor demand of firm i takes the form

$$\ell_i = h_i^* A_i. \tag{18}$$

Substituting eq. (18) in eq. (17) and using eq. (4) yields the equilibrium wage

$$w_i^* = \frac{y\varphi h_i^* \bar{h}_{\Gamma_i}^* k_i}{b + \varphi h_i^* \bar{h}_{\Gamma_i}^* k_i} = \frac{y\lambda L_i}{b + \lambda L_i}, \tag{19}$$

which explicitly shows that larger firms pay higher wages. In other words this result captures the the well-known employer size premium (Brown and Medoff, 1989; Brown et al., 1990). It also suggests that firms with a higher degree pay higher salaries when compared to other firms with the same h_i and $\bar{h}_{\Gamma_i}$.

Substituting eq. (19) in eq. (6) yields i's equilibrium hiring policy

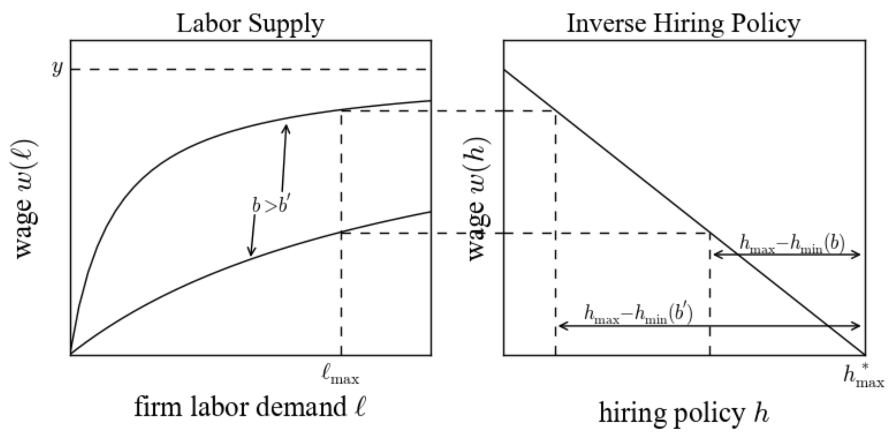
$$h_i^* = \min\left(1, \frac{\phi b - \sqrt{\phi^2 b^2 + \phi\psi\varphi 2by\bar{h}^*_{\Gamma_i} k_i}}{-2\phi\varphi\bar{h}^*_{\Gamma_i} k_i}\right), \tag{20}$$

where the firm sets either a fraction $h_i^* \geq 0$ or a corner solution where it hires all applicants. Note that eq. (20) in its vector form is continuous map $T : [0,1]^N \to [0,1]^N$. Therefore, a set $\{h_i^*\}_{i=1}^N$ exists.

Equation (20) captures captures the interaction between the hiring behavior of firm i (expressed through h_i) and the hiring behavior of its neighbors, correlating hiring policies across the LFN in a negative fashion. This has important implications on labor reallocation. For example, if a worker leaves a firm with a low hiring policy, his or her employment prospects will be limited to companies with a similar hiring rate. Therefore, escaping this cluster of poor employment prospects takes longer thank in a matching process where hiring policies are well-mixed across firms. This has a profound effect on our understanding of local shocks and unemployment traps because the former exacerbate these bottleneck effects, generating unemployment and traps. For instance, we know by eq. (4) that higher a k_i induces a larger firm size. Then, the negative correlation between k_i and h_i means that a larger proportion of workers (those in the largest firms) are searching for jobs in firms with lower hiring policies (their neighbors). Following this logic, we can expect that a LFN with a degree distribution that is a mean-preserving spread of another one induces higher a level of unemployment.

There is an important connection between the topology of the LFN and the optimal hiring policies. Its importance relies on the degree of heterogeneity of the set $\{h_i^*\}_{i=1}^N$. If there is a large spread of hiring policies, then the effect of the network topology on aggregate unemployment is larger. In this model, the diversity of hiring policies comes from the supply

Figure 2: Wage Dispersion and Hiring Policies



The left panel shows two aggregate labor supplies with different elasticities obtained from eq. (17). It also presents the corresponding wages that the firm with the largest demand $\ell_{\max}$ would have to pay when confronting each supply. The right panel maps these wages through eq. (6), into the hiring policies that would be set by the firm with the largest demand.

elasticity, because that is the main determinant of wage dispersion[2]. Figure 2 illustrates the relationship between supply elasticity, wages, and hiring policies. To build an intuition, consider the firm with the largest labor demand $\ell_{\max}$, which determines the maximum wage in the economy. The latter is higher in an economy with a more inelastic labor supply, considering everything else constant. A higher wage implies a lower hiring policy for this firm, increasing the dispersion between the maximum hiring policy $h_{\max}$ and the lowest one $h_{\min}$. Firms with different degrees set different hiring policies (assuming that $\bar{h}_{\Gamma_i}$ does not cancel the effect of k_i). Therefore, heterogeneity in both, wages and the topology of the LFN, are important. Both of these elements are present in empirical datasets. Therefore, this modeling framework seems adequate and points to important network effects that have not been previously accounted for.

In order to better understand the aggregation of unemployment, it is important to an-

[2]However, the model is flexible enough to allow firm heterogeneity in parameters such as the separation rate λ, the productivity y, the hiring cost c, and the sunk cost κ. This is an important strength of the model because it facilitates more realistic calibrations that consider the cross-sectional variation of firms.

alyze how the correlation of hiring policies interact with the topology of the LFN. For this purpose, we study three representative cases of well-known network topologies and then proceed to apply our model to the empirical topology obtained from our data.

4.1 Styized Networks

We are interested in learning how large are the effects of the LFN topology on aggregate unemployment, when hiring policies correlate according to eq. (20). For this purpose, we analyse the outcome of the model under three random networks that relate to homogeneous and heterogeneous job search processes. The first is a regular graph, i.e. a network where every firm has the same number of connections. The second is the popular Erdős-Rényi graph, where the firm degree follows a binomial distribution. The third is the so-called scale-free network, which has Pareto-distributed degrees. Naturally, there are other elements that define the topology of real-world network (e.g., clusters, path length, closeness, etc.). Here we focus on the degree to build an initial intuition, while in the **??** we concentrate on empirical LFNs.

Each of these stylized networks is differentiated by its degree heterogeneity. To study the effect induced by such difference, we have chosen topologies that have the same mean degree $\bar{k}$, i.e. it has a Dirac delta distribution. Therefore, Erdős-Rényi degree distribution is a mean-preserving spread of the regular graph, while the scale-free is a mean-preserving spread of the other two. In addition, processes that take place on the regular and the Erdős-Rényi graphs can be well-approximated by aggregations because the degree heterogeneity is negligible. This is not the case in for the scale-free network, so is is important that we study the differences produced by these three topologies. For the case of the regular graph, it is easy to obtain closed form solution of eq. (20) by substituting $\bar{h}^*_{\Gamma_i}$ by h^* in eq. (20) and using eq. (10). This yields

$$h^* = \frac{bN(y\psi\theta - 2\lambda\phi) + \sqrt{b^2N^2(2\lambda\phi + y\psi\theta)^2 + 8byNH\lambda^2\phi\psi\theta}}{4\phi\theta(bN + H\lambda)}, \tag{21}$$

where $\theta = 1-(1-v)^k$. For the case of the networks with heterogeneous degrees, the solutions are obtained numerically. A formal proof of the uniqueness of the the fixed point in eq. (20) is not straightforward. However, numerical experiments via Monte Carlo simulation suggest that eq. (20) is a contraction mapping, providing a consistent solution that we use to compute aggregate unemployment.

Panel A in fig. 3 shows the Beveridge curves generated by the model. Here, we portray the Beveridge curve as the relationship between the unemployment rate and the average hiring policy. The curves are generated by solving the model for different levels of the hiring cost c in the interval $[0.1, 0.9]$. Two notable features stand out in this diagram. First, the curve from the scale-free network is significantly distant from the other two. Second, the three curves collapse when $\bar{h}^* = 1$. This is quite intuitive when we consider the sampling process that workers undergo in the LFN. If all firms set hiring policies near 1, the likelihood of getting a job depends mostly on the investment shocks, which happen uniformly across firms. In this situation, a job seeker at a firm with few edges has almost the same chance of finding a job as a worker at a firm with many connections. This also relates to the dispersion of $\{h_i^*\}_{i=1}^N$ because when firms hire all applicants there is no diversity of hiring policies, which cancels out the effect of the LFN.

Panel B in fig. 3 shows the employer-size premium across the three networks. It is clear that the network with largest degree heterogeneity also has the largest wage dispersion. The topology of the network does not shift the $L-w$ curve so we cannot expect significant changes in the average wage due to network structure. Panel C demonstrates the interaction between firms' hiring behavior and their neighbors'. As suggested in eq. (20), there is a negative relationship between h_i^* and $\bar{h}_{\Gamma_i}^*$. These correlations are clustered by levels of h_i^* and their dispersion is larger in the scale-free network.

As shown in panel D of fig. 3, firms with more edges tend to set lower hiring policies. The mechanism is simple: with more neighbors, A_i grows and so does i's demand for labor. More demand implies a higher wage to be paid by the firm, which shifts its profit curve to the left. In order to compensate for higher salaries, the firm needs to re-adjust h_i^* to a lower level. Finally, as predicted by eqs. (4) and (9), firms with higher connectivity tend to be larger and have more associated unemployed agents. In addition, the network with a Pareto degree distribution also exhibits a larger firm size dispersion.

4.2 Empirical Networks

We would like to conclude by analyzing real-world LFNs and learning something about the empirical implications of their topologies. For this purpose, we calibrate the model to match the observed aggregate unemployment rates of Finland throughout 20 years, while controlling for its LFNs and separation rates. In order to estimate λ, we make use of our last theoretical result

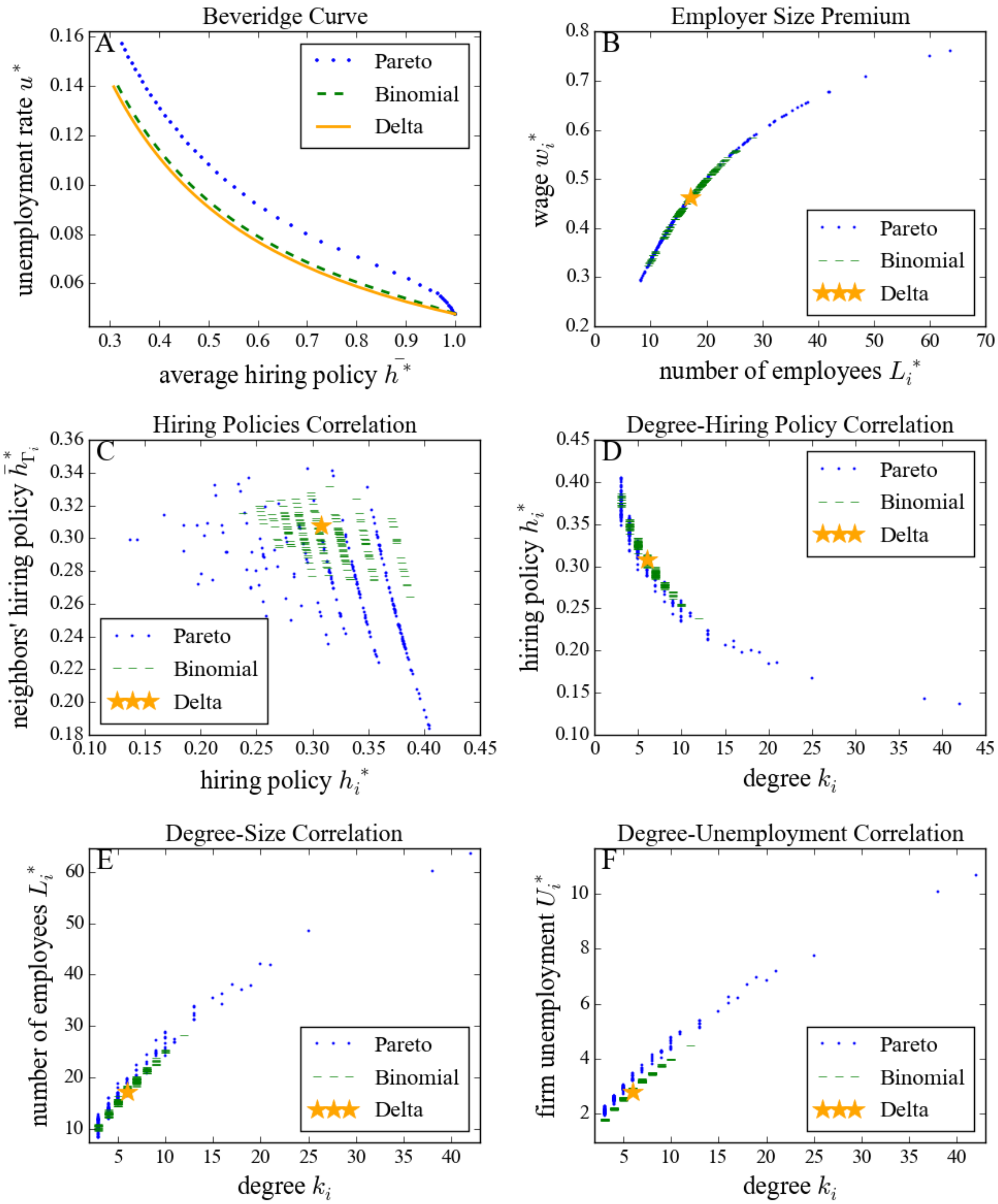
Proposition 7. *The steady-state average number of unemployed who become employed after being associated to a firm i that follows eqs.* (2) *and* (3) *is*

$$O_i = \varphi h_i \bar{h}_{\Gamma_i} k_i. \tag{22}$$

The proof follows from the fact that, in the steady-state, $O_i = \lambda L_i$ (see appendix). The intuition is simple: we can consider firm-specific unemployment as a pool of people that is constant through time. The inflows into U_i are λL_i while the outflows are O_i. In order for U_i to be constant, the inflows and the outflows must be equal.

Taking advantage of eq. (22), we use the steady-state condition $O_i = \lambda L_i$ in order to estimate the model

Figure 3: Equilibrium Outcomes on Different Network Topologies



Equilibrium solutions for an example calibration: $\{N = 200,\ H = 4000,\ \lambda = .05,\ y = 1,\ v = .8,\ c = .1,\ \kappa = .5,\ b = 1\ \}$, and different network topologies with the same average degree of 6. The solution for the network with a Dirac delta degree distribution was obtained through eq. (21), while the ones for the binomial and Pareto degree distributions were obtained numerically. Panel a shows the solutions for different levels of c. The rest of the panels show the cross-sectional variation of the solution for representative networks.

$$O_i = \beta_\lambda L_i + \epsilon_i, \tag{23}$$

where $\beta_\lambda = \lambda$. We calibrate the model to a daily frequency, so the estimated separation rate becomes $\hat{\beta}_\lambda^d = 1 - (1 - \hat{\beta}_\lambda)^{\frac{1}{365}}$ (see appendix).

During the calibration process, we want to avoid trivial solutions such as homogeneous sets of hiring policies. This is so because homogeneity misses important empirical regularities, for example, wage dispersion, heterogeneous firm sizes, and the employer-size premium. We use parameters c, κ, and b for this purpose. As previously discussed, b allows wage dispersion, so an inelastic labor supply is desirable in order to generate heterogeneous hiring policies. Parameter c determines the overall level of w_i, hence of h_i^*. Finally, κ limits the maximum w_i by making the firm more sensitive to the investment shocks, even when it is closed. We normalize $y = 1$ and allow v to be a degree of freedom to calibrate the model and match the observed level of aggregate unemployment.

Once calibrated, we use the model to compute a counter-factual. This counter-factual consists of evaluating the model under a different network structure, while keeping everything else constant. Put it differently, we estimate what would be the aggregate unemployment rate in Finland if the frictions of the labor market would have a homogeneous structure (an implicit assumption in aggregate job search models). In other words, we compute aggregate unemployment when $k_i = k$, which is given by eq. (13), where h^* corresponds to the solution of the homogeneous case in eq. (21). We perform this exercise for different supply elasticities in order to gain some insights about the minimum and maximum effects of the network topology.

Figure 4 shows the difference in aggregate unemployment between the fitted model and the counter-factual. We present results for three levels of supply elasticity[3]. As discussed previously, a more inelastic labor supply generates more wage dispersion, which contributes

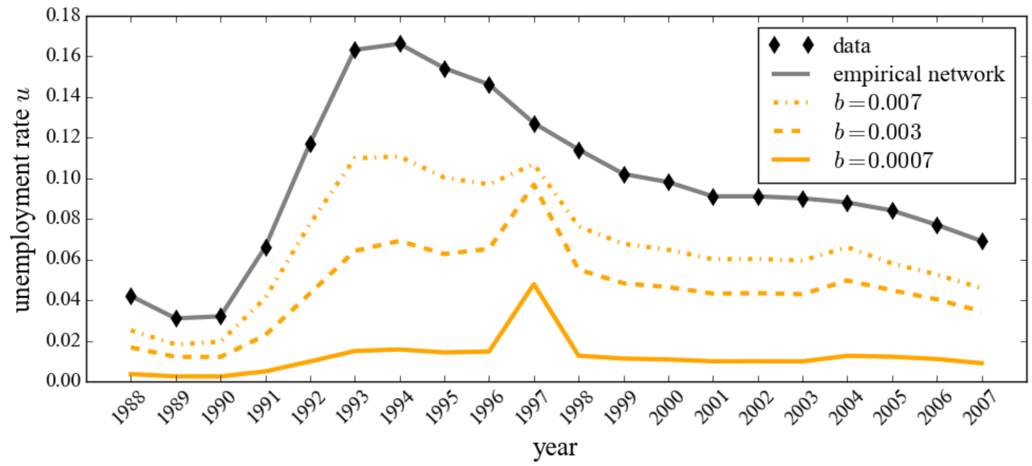
[3]The bump in the counter-factual of 1997 is caused by an anomaly in the data. Due to changes in data administration, 1997 registers a substantial increase in N (see table 1). Most of these firms have $k_i = 1$, so the average degree drops nearly 50% with respect to 1996.

to a larger difference in unemployment between the real LFN and the regular network. We interpret this difference as the contribution of the network structure to aggregate unemployment. Under a very elastic labor supply, the contribution is marginal. However, if the supply is highly elastic, the contribution of the network topology can account for more than 90% of the unemployment rate. Given that real economies exhibit wage dispersion, the LFN is likely to have a significant effect on aggregate unemployment.

Naturally, any aggregate model (implying a regular network) could also be calibrated to match the empirical level of unemployment. Then, the counter-factual of a heterogeneous network structure yields a higher unemployment rate. The important point in this exercise is that, if one would like to predict unemployment after a change in parameters, it is likely that the aggregate model will underestimate the change in unemployment because the underlying homogeneous structure is less sensitive. Furthermore, the heterogeneous structure of the LFN is observed from empirical microdata on how labor is actually reallocated, something omitted when aggregating the matching process. Therefore, further investigations in the direction of job search on networks would be desirable in order to better understand labor dynamics and the limitations of aggregate approaches.

Finally, the LFN topology not only affects the level of aggregate unemployment, but also its variation through time. In this exercise, it is evident that more degree heterogeneity also increases the magnitude of annual variations of the unemployment rate. This is an important result considering that the origins of unemployment volatility is a highly debated topic (Mortensen and Nagypál, 2007; Pissarides, 2009; Shimer, 2010; Obstbaum, 2011). If structural changes or shocks take place (e.g., changes in λ or v), the labor reallocation process is smoother on a regular structure than on a heterogeneous one. This is quite intuitive when thinking in terms of job search as a gradual navigation on a network. A shock or a structural change generates heterogeneous adjustments of hiring policies when the network is not regular (and assuming wage dispersion). If the LFN has firms that concentrate

Figure 4: Equilibrium Unemployment and Counterfactuals



The diamonds correspond to the observed annual aggregate unemployment rate. The grey line was obtained by calibrating the model to match the observed unemployment rates of each year using parameter values: $y = 1$, $c = .1$, $\kappa = .5$, and $H = 2,000,000$ (the size of the Finnish labor force). N is the number of firms in the data, λ was estimated form the data, and v varies between years due to the fitting procedure.

many connections, labor reallocation becomes susceptible to the congestion effects that these companies generate by re-adjusting their hiring policies. In a regular topology the reallocation process is smoother because the shock or structural change generates the same re-adjustment across all firms, which happens to have the same number of employees and associated unemployed. Therefore, the LFN points towards the need to understand the propagation of shocks and structural changes through the gradual reallocation of labor that takes place on the network, something that we leave for future work.

5 Discussion and Conclusion

We developed a framework to study aggregate unemployment from new network-theoretic micro-foundations of job search as a gradual navigation process on a LFN. The framework allows to study the composition of aggregate unemployment with a resolution at the level of each firm. It also shows that an externality emerges between neighbor firms: ‘my growth affects yours. We found that when labor is reallocated through networks with degree heterogeneity, hiring policies correlate negatively through the LFN. Depending on the elasticity of the labor supply, the model generates wage dispersion and the topology of the LFN contributes in a significant way to the level of aggregate unemployment. This means that the way in which labor market frictions are structured (the network topology) plays a central role in the process of labor reallocation because this structure reshapes the pathways that labor uses to navigate through different firms. Through their hiring behavior, firms modulate the flows of labor, generating pockets of local unemployment and congestion effects. This framework provides a rich and elegant, description of decentralized labor markets with the possibility of preserving important information that is lost through aggregate approaches.

Our theory is empirically supported by comprehensive micro-data on employer-employee matched records. It suggests that the role of firm connectivity is key to link individual firm dynamics to aggregate unemployment. Moreover, we found that, in the case of Finland, the structure of the LFN may account for most of the aggregate unemployment rate. The framework also provides a new way to estimate separation rates and hiring policies. In addition, our results suggest that the collection of new information such as firm-specific unemployment could be useful to complement our knowledge about aggregate unemployment and the role of labor policy. For example, it could shed new light on the origins of unemployment volatility and mismatch unemployment.

On the theoretical side, the LFN framework can be employed to consider firm-specific phenomena. In addition, this framework is particularly well suited to study the propagation of local shocks and structural changes, a major issue in labor policy discussions. Its localized nature allows it to be implemented through other methods such as computer simulation and agent-computing models (Freeman, 1998; Geanakoplos et al., 2012) in order to study the impact and timing effects of specific policies. This facilitates the study of a richer set of problems that are difficult to address from an aggregate perspective. For example, we could use employer-employee matched records to calibrate an agent-computing model with the real LFN and then simulate local shocks to groups of firms. The computational model would allow us to obtain information about how labor would flow out of the affected parts of the economy, and gradually find its way to firms with better employment prospects. Characterizing this gradual navigation process would be extremely helpful in designing policies that aim not only to alleviate unemployment, but to smooth transitional phases of the economy.

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Appendix (For Online Publication)

Proof of proposition 1

Let $p_i(t)$ and $q_i(t)$ be the probabilities of being employed and unemployed at firm i in period t respectively. Both quantities are dynamically described by

$$p_i(t) = (1-\lambda)p_i(t-1) + h_i \sum_{j\in\Gamma_i} q_j(t-1) \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) \frac{1}{\left|\gamma_j^{(i)}\right|}, \tag{24}$$

and

$$q_i(t) = \lambda p_i(t-1) + q_i(t-1)\left[\sum_{\gamma\neq\emptyset} \Pr(\gamma_i)\frac{1}{|\gamma_i|}\sum_{j\in\gamma_i}(1-h_j) + \Pr(\gamma_i=\emptyset) + (1-s)\right], \tag{25}$$

where $\gamma_j^{(i)}$ indicates a configuration of open and closed neighbors of j, such that i is open. The symbol $\{\gamma_j^{(i)}\}$ denotes the set of all possible configurations of open and closed neighbors of j where i is open. The set γ_i contains all open neighbors of i, and we denote $\emptyset$ the set of neighbors of i when all of them are closed.

In the steady-state, $p_i(t) = p_i(t-t) = p_i$ and $q_i(t) = q_i(t-t) = q_i$. Note that $\sum_{\gamma\neq\emptyset}\Pr(\gamma_i) + \Pr(\gamma_i=\emptyset) = 1$, so the system defined by eqs. (24) and (25) becomes

$$0 = -\lambda p_i + h_i \sum_{i\in\Gamma_i} q_j \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) \frac{1}{\left|\gamma_j^{(i)}\right|} \tag{26}$$

$$0 = \lambda p_i - q_i \sum_{\gamma\neq\emptyset} \Pr(\gamma_i)\bar{h}_{\Gamma_i}. \tag{27}$$

From eq. (27), let us write q_i in terms of p_i as

$$q_i = \frac{\lambda}{s\sum_{\gamma\neq\emptyset}\Pr(\gamma_i)\bar{h}_{\Gamma_i}} p_i, \tag{28}$$

and then substitute p_i with eq. (26) to obtain

$$q_i = \sum_{i\in\Gamma_i} \frac{q_j h_i \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) / \left|\gamma_j^{(i)}\right|}{\sum_{\gamma\neq\emptyset}\Pr(\gamma_i)\bar{h}_{\Gamma_i}}, \tag{29}$$

To understand this further, we write the previous equation in matrix form making use of the adjacency matrix of the graph, $\mathbf{A}$, for which $\mathbf{A}_{ij} = \mathbf{A}_{ji} = 1$ if i and j have an edge connecting them, and zero otherwise. This produces the expression

$$\sum_{j=1}^{N}\left[\mathbf{A}_{ij}\frac{h_i\sum_{\{\gamma_j^{(i)}\}}\Pr\left(\gamma_j^{(i)}\right)/\left|\gamma_j^{(i)}\right|}{\sum_{\gamma\neq\emptyset}\Pr(\gamma_i)\bar{h}_{\Gamma_i}}-\delta[i,j]\right]\lambda p_j=0 \tag{30}$$

for all i. This represents a homogeneous system of linear equations, which always has the trivial null solution, and has non-trivial solutions if and only if the matrix contained inside brackets is singular which, among other things, implies that the matrix does not have full rank. To show that our model has non-trivial solutions indeed, we define the matrix $\mathbf{\Lambda}$, with element $\mathbf{\Lambda}_{ij}$ corresponding to the expression inside brackets

$$\mathbf{\Lambda}_{ij}:=\mathbf{A}_{ij}\frac{h_i\sum_{\{\gamma_j^{(i)}\}}\Pr\left(\gamma_j^{(i)}\right)/\left|\gamma_j^{(i)}\right|}{\sum_{\gamma\neq\emptyset}\Pr(\gamma_i)\bar{h}_{\Gamma_i}}-\delta[i,j]. \tag{31}$$

This matrix does not possess full rank as can be explicitly seen from the fact that all columns add to zero. To show this, we first sum $\mathbf{\Lambda}_{ij}$ over i

$$\sum_{i=1}^{N}\mathbf{\Lambda}_{ij}=-1+\sum_{i=1}^{N}\mathbf{A}_{ij}\frac{h_i\sum_{\{\gamma_j^{(i)}\}}\Pr\left(\gamma_j^{(i)}\right)/\left|\gamma_j^{(i)}\right|}{\sum_{\gamma\neq\emptyset}\Pr(\gamma_i)\bar{h}_{\Gamma_i}} \tag{32}$$

where -1 comes from $-\sum_i\delta[i,j]$. We can now show that the numerator and denominator of the second term are indeed equal. To see this in detail, we organize the elements of $\{\gamma_j^{(i)}\}$ by cardinality $|\gamma_j^{(i)}|$, and rewrite the numerator as

$$\sum_{i=1}^{N}\mathbf{A}_{ij}h_i\sum_{\{\gamma_j^{(i)}\}}\Pr(\gamma_j^{(i)})/|\gamma_j^{(i)}|=\sum_{c=1}^{|\Gamma_j|}\frac{1}{c}\sum_{i}\mathbf{A}_{ij}h_i\sum_{|\gamma_j^{(i)}|=c}\Pr(\gamma_j^{(i)}), \tag{33}$$

where the last sum is over all elements of $\{\gamma_j^{(i)}\}$ with equal size c. Now, the sum over i guarantees that each neighbor of j belonging to a particular $\gamma_j^{(i)}$ is summed, along with the corresponding h_r, where $r\in\gamma_j^{(i)}$. Therefore, the sum over i can be rewritten as

$$\sum_{i}\mathbf{A}_{ij}h_i\sum_{|\gamma_j^{(i)}|=c}\Pr(\gamma_j^{(i)})=\sum_{|\gamma_j|=c}\left(\sum_{r\in\gamma_j}h_r\right)\Pr(\gamma_j) \tag{34}$$

and inserting this into the sum over c leads to

$$\sum_{c=1}^{|\Gamma_j|}\frac{1}{c}\sum_{|\gamma_j|=c}\left(\sum_{r\in\gamma_j}h_r\right)\Pr(\gamma_j)=\sum_{\gamma_j\neq\emptyset}\frac{\sum_{r\in\gamma_j}h_r}{|\gamma_j|}\Pr(\gamma_j)=\sum_{\gamma_j\neq\emptyset}\langle h\rangle_{\gamma_j}\Pr(\gamma_j) \tag{35}$$

Therefore,

$$\sum_{i=1}^{N} \mathbf{A}_{ij} h_i \sum_{\{\gamma_j^{(i)}\}} \Pr(\gamma_j^{(i)})/|\gamma_j^{(i)}| = \sum_{\gamma_j \neq \emptyset} \langle h \rangle_{\gamma_j} \Pr(\gamma_j) \tag{36}$$

which means that for all j, eq. (32) is identically zero, guaranteeing that the system has non-trivial solutions.

Since the matrix for a connected graph has rank $N-1$, its kernel is one-dimensional, and thus, to choose a unique solution that belongs to the kernel of $\mathbf{\Lambda}$ we need a single additional condition. In our case, this condition corresponds to

$$\sum_{i=1}^{N} (p_i + q_i) = 1, \tag{37}$$

which guarantees that each individual is either employed or unemployed and associated to only one firm each period. *Q.E.D.*

Proof of proposition 2

Let us consider eqs. (26) and (27) and note that the probability $\Pr(\gamma_i)$ of obtaining a specific configuration γ_i of open and closed neighbors follows the binomial $v^{|\gamma_i|}(1-v)^{k_i-|\gamma_i|}$. Then, we obtain that

$$\sum_{\{\gamma_j^{(i)}\}} \Pr(\gamma_j^{(i)})/|\gamma_j^{(i)}| \to \sum_{|\gamma_j^{(i)}|=1}^{k_j} \binom{k_j-1}{|\gamma_j^{(i)}|-1} \frac{v^{|\gamma_j^{(i)}|}(1-v)^{k_j-|\gamma_j^{(i)}|}}{|\gamma_j^{(i)}|} = \frac{1-(1-v)^{k_j}}{k_j}. \tag{38}$$

For the sum $\sum_{\gamma_j \neq \emptyset} \bar{h}_{\Gamma_i} \Pr(\gamma_j)$, we note that each hiring policy h_i for $i \in \Gamma_j$ appears $\binom{k_j-1}{|\gamma_j|-1}$ times among all the terms where there are $|\gamma_j|$ open neighbors to j. We can then write

$$\sum_{\gamma_j \neq \emptyset} \bar{h}_{\Gamma_i} \Pr(\gamma_j) \to \sum_{|\gamma_j|=1}^{k_j} \binom{k_j-1}{|\gamma_j|-1} \frac{\sum_{i\in\Gamma_j} h_i}{|\gamma_j|} v^{|\gamma_j|}(1-v)^{k_j-|\gamma_j|} = \bar{h}_{\Gamma_i}(1-(1-v)^{k_j}), \tag{39}$$

where $\bar{h}_{\Gamma_i} := \sum_{i\in\Gamma_j} h_i/k_j$, i.e., the average hiring policy of the full neighbor set of j. Therefore, eqs. (26) and (27) simplify into

$$0 = -\lambda p_i + h_i \sum_{i\in\Gamma_i} q_j \frac{1-(1-v)^{k_j}}{k_j} \tag{40}$$

$$0 = \lambda p_i - q_i \bar{h}_{\Gamma_i}[1-(1-v)^{k_i}]. \tag{41}$$

It is easy to see by inspection that the solution to the system is

$$p_i = \frac{\chi h_i \bar{h}_{\Gamma_i} k_i}{\lambda} \tag{42}$$

$$q_i = \frac{\chi h_i k_i}{1-(1-v)^{k_i}} \tag{43}$$

$$\chi = \frac{1}{\sum_i h_i \bar{h}_{\Gamma_i} k_i \left[\frac{1}{\lambda} + \frac{1}{\bar{h}_{\Gamma_i}[1-(1-v)^{k_i}]}\right]}. \tag{44}$$

Given that the workers' actions are independent from each other, the evolution of the firm size follows the binomial

$$\Pr(L_i) = \binom{H}{L_i} p_i^{L_i} (1 - p_i)^{H - L_i}, \tag{45}$$

so the steady-state average firm size L_i (abusing notation) is

$$L_i = H p_i = \frac{\varphi h_i \bar{h}_{\Gamma_i} k_i}{\lambda}, \tag{46}$$

where $\varphi = H\chi$. *Q.E.D.*

Proof of proposition 3

Consider the probability $a_i(t)$ that a worker submits a job application to firm i in period t. This depends on: the probability $q_j(t-1)$ of being unemployed in a neighbor $j \in \Gamma_i$ during the previous period; on the probability $\Pr(\gamma_j^{(i)})$ of j having a configuration $\gamma_j^{(i)}$ of open of closed neighbors in which i is open; and on the probability of choosing i over all other alternative neighbors of j. Accounting for all possible events and configurations of neighbors, this probability is written as

$$a_i(t) = \sum_{j \in \Gamma_i} q_j(t-1) \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) \frac{1}{\left|\gamma_j^{(i)}\right|}. \tag{47}$$

In the steady-state $a_i(t) = a_i(t-1) = a_i$ and $q_i(t) = q_i(t-1) = q_i$, and by replacing eqs. (38) and (43) we obtain

$$a_i = \chi \bar{h}_{\Gamma_i} k_i. \tag{48}$$

Since the workers' behaviors are independent from each other, the number of job applications received by firm i in any period follows the binomial

$$\Pr(A_i) = \binom{H}{A_i} a_i^{A_i} (1-a_i)^{H-A_i}, \tag{49}$$

so the steady-state average number of applications A_i (abusing notation) is

$$A_i = H a_i = \varphi \bar{h}_{\Gamma_i} k_i, \tag{50}$$

where $\varphi = H\chi$. A_i fulfills the steady-state balance condition $\lambda L_i = h_i A_i$. *Q.E.D.*

Proof of proposition 4

Let us consider the steady-state solution for the probability q_i of being unemployed and associated to firm i, as written in eq. (43). Given that the workers' actions are independent from each other, the evolution of the firm-specific unemployment follows the binomial

$$\Pr(U_i) = \binom{H}{U_i} q_i^{U_i} (1 - q_i)^{H - U_i}, \tag{51}$$

so the steady-state average firm-specific unemployment U_i (abusing notation) is

$$U_i = H q_i = \frac{\varphi h_i k_i}{1 - (1 - v)^{k_i}}, \tag{52}$$

where $\varphi = H\chi$. *Q.E.D.*

Proof of proposition 7

Consider the probability $o_i(t)$ that a worker associated to firm i finds a job at a different firm in period t. This event depends on: the probability $q_i(t-1)$ that the worker was unemployed and associated to firm $i \in \Gamma_j$ during the previous period, on the probability $\Pr(\gamma_i)$ of i having a configuration γ_i of open of closed neighbors; and on the probability of choosing one particular firm over all other alternatives available in Γ_i. Altogether, these factors constitute probability

$$o_i(t) = q_i(t-1) \sum_{\gamma_i \neq \emptyset} \Pr(\gamma_i) \frac{1}{|\gamma_i|}. \tag{53}$$

In the steady-state $o_i(t) = o_i(t-1) = o_i$ and $q_i(t) = q_i(t-1) = q_i$, and by replacing eqs. (39) and (43) we obtain

$$o_i = \chi h_i \bar{h}_{\Gamma_i} k_i. \tag{54}$$

Since the workers' behaviors are independent from each other, the number of i's outflows in any period follows the binomial

$$\Pr(O_i) = \binom{H}{O_i} o_i^{O_i} (1 - o_i)^{H - O_i}, \tag{55}$$

so the steady-state average outflows O_i (abusing notation) is

$$O_i = H o_i = \varphi h_i \bar{h}_{\Gamma_i} k_i, \tag{56}$$

where $\varphi = H\chi$. O_i fulfills the steady-state balance condition $O_i = \lambda L_i$. *Q.E.D.*

Estimation of Separation Rates for Finland

Year	β_{λ}	N	R^2
1988	0.188*** (2.900e-02)	34,279	0.407
1989	0.102*** (2.224e-02)	32,771	0.301
1990	0.105*** (2.442e-02)	25,260	0.246
1991	0.049*** (9.445e-03)	19,143	0.252
1992	0.028*** (4.575e-03)	16,810	0.141
1993	0.188 (1.270e-01)	17,667	0.174
1994	0.150* (7.513e-02)	20,756	0.279
1995	0.068*** (1.759e-02)	21,012	0.151
1996	0.059*** (6.019e-03)	24,076	0.382
1997	0.065*** (8.652e-03)	51,493	0.509
1998	0.088*** (1.590e-02)	31,322	0.281
1999	0.208* (8.141e-02)	33,648	0.409
2000	0.154** (4.993e-02)	34,008	0.342
2001	0.088*** (1.667e-02)	33,331	0.323
2002	0.066*** (1.103e-02)	33,031	0.376
2003	0.070*** (1.041e-02)	33,842	0.367
2004	0.592*** (1.528e-01)	35,924	0.609
2005	0.074*** (1.029e-02)	41,978	0.415
2006	0.127*** (3.286e-02)	44,403	0.524
2007	0.086*** (1.171e-02)	42,767	0.470

Table 3: Estimation of annual separation rates for Finland via eq. (23). Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.