Partner Uncertainty and the Dynamic Boundary of the Firm*

Thomas Hellmann† and Veikko Thiele‡

January 2016

Forthcoming in the *American Economic Journal: Microeconomics*

Abstract

We develop a new theory of the dynamic boundary of the firm where asset owners may want to change partners ex-post. The model identifies a fundamental trade-off between (i) a "displacement externality" under non-integration, where a partner leaves a relationship even though his benefit is worth less than the loss to the displaced partner, and (ii) a "retention externality" under integration, where a partner inefficiently retains the other. With more asset specificity, displacement externalities matter more and retention externalities less, so that integration becomes more attractive. Our model also shows that wealthy partners would want to commit to ex-post wealth constraints.

Keywords: Asset ownership, control rights, firm boundaries, asset specificity, specific investments, wealth constraints.

JEL classification: D23, D82, D86.

*An earlier version of this paper was circulated under the title "A Theory of the Firm based on Partner Displacement." We would like to thank Philippe Aghion, Florian Ederer, Bob Gibbons, Trond Olsen, Barbara Spencer, Birger Wernerfelt, Ralph Winter, seminar participants at Texas A&M University, Carnegie Mellon University, the University of British Columbia, the University of Toronto, and Drexel University, and participants of the European Meeting of the Econometric Society (ESEM) in Malaga, the Annual Meeting of the European Association for Research in Industrial Economics (EARIE) in Rome, and the Annual Conference of the Canadian Economics Association in Vancouver for valuable comments and suggestions.

†University of Oxford, Saïd Business School, Park End Street, Oxford OX1 1HP, United Kingdom, e-mail: thomas.hellmann@sbs.ox.ac.uk.

‡Queen's University, Smith School of Business, Goodes Hall, 143 Union Street, Kingston, Ontario, Canada K7L 3N6, e-mail: thiele@queensu.ca.

1 Introduction

A central question in the theory of the firm is who should own the productive assets. It is commonly presumed that all asset owners know who their optimal trading partners are. In this paper we introduce uncertainty about the optimal partner match. This uncertainty generates a dynamic trade-off between the commitment to a trading relationship (integration) versus the flexibility of seeking new relationships (non-integration). We ask how partner uncertainty affects the allocation of property rights over productive assets, and how it influences the subsequent evolution and performance of the firm.

We identify a novel trade-off between integration and non-integration that is based on the dynamics of partner changes. Non-integration gives parties the freedom to easily leave their partners, whereas integration gives parties the security that their partners cannot easily leave them. Each regime has its strengths and weaknesses. Under non-integration parties have the flexibility of leaving, but may also find themselves in a situation where the harm to the party left behind exceeds the benefit to the leaving party. We call this a *displacement externality*. Under integration no such inefficient leaving occurs, but partners may find themselves in the opposite situation: Partners may inefficiently stay together even though the individual value of leaving would exceed the joint value of staying together. We call this a *retention externality*.

We use a model set-up similar to Grossman and Hart (1986) where there are two owner-managers. They have inalienable human capital, as well as alienable co-specialized assets, which we can think of physical capital or intellectual property. We allow for team production with private efforts, so that the partners' profit sharing agreement affects their effort incentives. The optimal allocation of property rights either consists of integration with joint asset control, or non-integration where each partner retains control over his own asset. We first consider a simple model without specific investments. This shuts down the standard trade-off for the optimal asset ownership known from the property rights theory, and allows us to focus on the new determinants that emerge solely from partner uncertainty.

The optimal ex-ante allocation of asset ownership depends on the degree of partner uncertainty, and the associated inefficiencies. Our base model shows that joint asset ownership is optimal when displacement externalities loom large, whereas individual asset ownership is preferred when retention externalities matter more. The relative importance of displacement and retention externalities depends on how good the original match between partners is. The greater the asset specificity, the greater the displacement externality, and also the smaller the retention externality. Higher asset specificity therefore favors joint asset ownership.

When allowing for relation-specific investments we find that joint asset ownership always provides stronger incentives for specific investments. The key intuition is that joint asset ownership is efficient when the internal match is good, but can cause retention externalities when the internal match is poor. By contrast, individual asset ownership is efficient when the internal match is poor, but can cause displacement externalities when the internal match is good. Consequently, joint (individual) asset ownership increases (decreases) the difference between the good and the bad match, which is good (bad) for incentives.

We also find that wealthy owners actually want to constrain the amount of wealth that is available for ex-post transfers. This is because having wealth is a double-edged sword: On the one hand, it enables transfer payments that mitigate ex-post inefficiencies. On the other hand, it weakens ex-ante incentives for specific investments, precisely because it allows partners to mitigate ex-post inefficiencies when the partner match is poor. In fact, the model shows that the optimal wealth is always sufficiently low to create a binding constraint.

The model generates testable predictions concerning the dynamics of firm boundaries and partner selection. Most important is the prediction that integration is associated with greater partner stability. This prediction stands out against the property rights literature (Grossman and Hart, 1985) which typically assumes optimal partner matches, and therefore does not even consider dynamic stability. The model also generates predictions about the initial allocation of property rights over assets. The higher the quasi-rents for the initial partner match, the more desirable it is to integrate. This formalizes an argument commonly associated with transaction cost theories (Williamson, 1985), namely that integration is more attractive when the value of an asset within the partnership is high compared to its current outside value. However, the model also predicts that the higher the expected quasi-rents in a potential match with an alternative partner, the less attractive it is to integrate upfront. This last prediction takes a new dynamic perspective, comparing current quasi-rents against potential future quasi-rents from alternative trading relationships.

2 Related Literature

Our model makes several departures from the seminal property rights models of Grossman, Hart and Moore (GHM henceforth).[4] First, our base model deliberately excludes specific in-

[4]See Grossman and Hart (1986), Hart and Moore (1990), and Hart (1995).

vestments, which is the central mechanism for determining asset ownership in the GHM model.[5] Second, our model allows for ex-post inefficiencies, which do not occur in GHM. Third, in the GHM model switching to an outside partner is a threat that is never exercised in equilibrium, whereas in our model partner changes actually occur in equilibrium. For example, buyouts can actually occur in our model. Fourth, our model allows partners to contractually specify prices ex-ante.[6] Fifth, in our model the optimal type of integration is joint asset ownership, whereas in the GHM model integration always consists of one agent owning both assets.[7]

Our theory provides a fresh perspective on one of the central tenets of transaction cost economics. Williamson (1975, 1985) argues that higher asset specificity should lead to integration, providing some verbal reasoning about opportunism and ex-post price haggling. More formal theories tend to dismiss these explanations, because rational agents should be able to resolve ex-post inefficiencies, and anticipate ex-ante any distributional consequences.[8] Yet, there is strong empirical support that asset specificity is associated with integration.[9] In our model binding wealth constraints create ex-post inefficiencies that are robust to renegotiation. Asset specificity matters not because of price haggling, but because of partner uncertainty, and its associated displacement and retention externalities. We also augment the standard transaction cost logic by identifying a dynamic trade-off, namely that asset specificity in the current relationship has to be compared against expected asset specificity in a potential future relationship.

A prior literature considers the possibility of ex-post inefficiencies.[10] Of historic interest is that, in addition to their seminal 1986 paper, Grossman and Hart published a less well known

[5]Note, however, that our model includes private effort. We incorporate a moral-hazard-in-teams problem (Holmström, 1982) into our production function. This yields a concave utility frontier as long as the agents' wealth constraints are binding, which generates the ex-post inefficiencies.

[6]The non-contractibility of prices is crucial for the property rights theory. We assume that prices are contractible at all times. However, our model does have some contractual incompleteness concerning interim information that allows partners to update their profitability forecasts. If these updates are verifiable, then the optimal allocation of assets becomes state-contingent. Even then the underlying trade-off between displacement and retention externalities remains valid.

[7]Cai (2003) examines a model with both specific and general investments, and shows that joint asset ownership becomes optimal when the two types of investments are substitutes. Halonen (2002) provides conditions under which joint asset ownership is optimal in a repeated game framework; see also Blonski and Spagnolo (2003). Our model provides a novel reason for the optimality of joint asset ownership, namely to prevent the dissolution of efficient partnerships.

[8]Indeed, GHM's property rights theory challenges Williamson's reasoning, arguing that what matters are marginal incentives to increase asset specificity through specific investments (see also Whinston, 2003). More recently, several paper develop formal models with costly ex-post adjustments, in the spirit of the transaction cost literature. See in particular Bajari and Tadelis (2001), Tadelis (2002), Matouschek (2004), and Casas-Arce and Kittsteiner (2011).

[9]See Lafontaine and Slade (2007) for a comprehensive survey of the empirical literature.

[10]Gibbons (2005) identifies these as adaptation-based theories of the firm. Segal and Whinston (2012) classify them as theories with imperfect bargaining.

book chapter in 1987 with a model where there are ex-post inefficiencies and no specific investments (Grossman and Hart, 1987). More recently, Hart (2009) and Hart and Holmström (2010) examine asset ownership in models with "reference points" where in certain states agents can commit to inefficiently withhold cooperation without renegotiation. Aghion et al. (2012) provide a model where renegotiation is hampered by ex-post asymmetric information. They show how the ex-ante asset allocation plays a role over and above any contractual arrangements.

In our model ex-post inefficiencies derive from a binding wealth constraint. We are not the first to consider wealth constraints. Aghion and Bolton (1992), for example, use them in a financial contracting model. In their model there are fixed non-transferable private benefits that can lead to ex-post inefficient decisions, depending on the allocation of control rights. The main difference to our model is that they focus on financial structures in a single asset model with liquidation, whereas we consider integration decisions in a model with two assets and partner uncertainty.

Joint asset ownership in our model can also be interpreted as a set of mutually exclusive contracts. As such our paper is related to the large literature on exclusive contracting and vertical foreclosure.[11] Aghion and Bolton (1987) examine how a seller can lock buyers into long-term contracts to reduce the threat of entry from a competing seller. Bolton and Whinston (1993) use a property-rights approach to study how concerns about supply assurances can motivate vertical integration. Segal and Whinston (2000) show that exclusive contracts have no effect on specific investments. De Fontenay, Gans, and Groves (2010) further generalize these results. These models typically find that exclusive contracts matter if there is no renegotiation, but that they no longer matter once renegotiation is allowed. In our model there is renegotiation; yet exclusive contracts still matter because renegotiation cannot always achieve the efficient outcome.

Our paper is also related to the emerging literature on the economics of entrepreneurship. One part of this literature examines the formation of partnerships and teams. Prat (2002) considers the benefits of forming heterogeneous teams. Franco, Mitchell, and Vereshchagina (2011) identify conditions under which moral hazard leads to assortive matching among team members. Hellmann and Perotti (2011) examine how idea generators are matched with idea developers, both within firms and markets. These theories are mostly concerned with the process by which initial partners form a team. Hellmann and Thiele (2015) further ask at what stage founders actually want to commit to a team. Their theoretical set-up is related to the current model, but their focus is on relating the timing of contracting to uncertainty about founder skills.

[11] See also Jing and Winter (2014) for a broader overview of the literature on exclusionary contracts.

Our model takes some inspiration from the economic literature of marriage and divorce. The work of Brien, Lillard and Stern (2006) and of Matouschek and Rasul (2008) examine the consequences of changes in the cost of divorce. Peters and Siow (2002), Peters (2007), and Wickelgren (2009) examine the role of wealth and investments in marriage markets. Peters (1986), Friedberg (1998), Wolfers (2006), look at the effect of unilateral divorce law on the marriage relationship. Our model also involves the matching and re-matching of partners, and it also examines the roles of wealth and investments. However, our model is also clearly different from those marriage models. For example, we determine 'divorce' costs endogenously, and the role of wealth is entirely different in our model. We develop our model from the ground up, in order to directly address those issues that are relevant in the theory of the firm.[12]

The remainder of this paper is structured as follows. The next section introduces our main model. Section 4 examines how partners make choices about staying versus leaving a relationship, and identifies the optimal asset ownership in the absence of specific investments. In Section 5 we then analyze the role of asset ownership for the partners' incentives to make relation-specific investments. In Section 6 we identify the optimal allocation of control rights over critical assets, accounting for specific investments and ex-post transfer payments. In Section 7 we discuss how allowing for asymmetric partners would affect our main insights. Section 8 summarizes our main results, and explores avenues for future theoretical and empirical work. All proofs are in the Online Appendix, which is available on the authors'websites.

3 The Base Model

Consider an initial match of two risk-neutral partners, for ease of exposition called *Alice* (A) and *Bob* (B). For example, Bob can be the owner an upstream firm selling an input to Alice as the owner of a downstream firm, which Alice needs to manufacture an end product. The value of their initial outside options is normalized to zero. Each partner initially owns a co-specialized asset, and has wealth $w \equiv w_A = w_B \geq 0$.

There are five dates; see Figure 1 for a graphical overview. At date 0, both partners decide on an ownership structure for both assets. While we consider all ownership structures, the key decision will be whether partners keep individual asset ownership, or they agree on joint asset

[12] In this context it is also worth mentioning the small literature on dissolutions of partnerships, which presumes that there is prior joint ownership, but that the partners now want to dissolve their partnership. This literature focuses on information asymmetries across partners, and examines alternative auction mechanisms for how to dissolve the partnership. See in particular the work Crampton, Gibbons and Klemperer (1987), McAfee (1992), and de Frutos and Kittsteiner (2008).

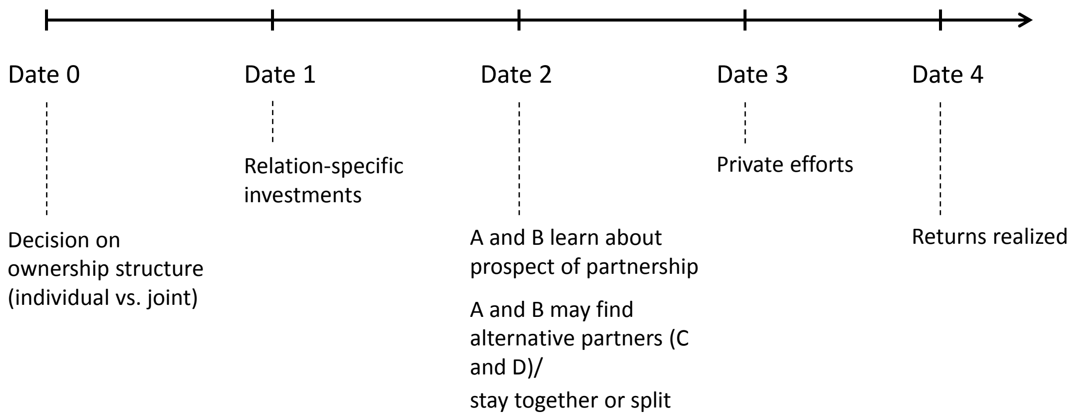

Figure 1: Timeline

ownership.[13] At date 1, both partners can make relation-specific investments to improve the value of joint production. At date 2, Alice and Bob learn about the prospect of their partnership, and may find alternative partners. They then decide whether to stay together, or to leave and form a new partnership. Alice and Bob may also renegotiate any division of surplus. At date 3, partners exert private effort to produce output. Finally, at date 4, all returns are realized.

In case of a successful joint production, Alice and Bob generate the profit y at date 4. We assume that y is verifiable, and that it has a distribution $\Omega_{in}(y)$ over some interval $y \in [\underline{y}, \overline{y}]$ with $0 \leq \underline{y} < \overline{y} \leq \infty$. We denote the expected value by $\pi = \int_{\underline{y}}^{\overline{y}} y d\Omega_{in}(y)$, and refer to it as the *inside prospect* of the match between Alice and Bob. We assume that the inside prospect π is observable by both partners, but non-verifiable by outside parties.

At date 1 Alice and Bob can invest in their relationship to improve the distribution of potential profits y. Specifically we assume that the expected profit π can take on two values: $\pi \in \{\pi_L, \pi_H\}$, with $\pi_H > \pi_L > 0$. The inside prospect π will be high ($\pi = \pi_H$) with probability $p = p(r_A, r_B)$, and low ($\pi = \pi_L$) with probability $1 - p$, where p is concave increasing in the partners' relation-specific investments r_A and r_B. Specific investments are non-contractible, and impose convex private costs $\psi(r_i)$, $i = A, B$, with $\psi(0) = \psi'(0) = 0$. To ensure interior solutions we assume that $p(0,0) = 0$ and $\partial p(\cdot)/\partial r_i|_{r_i=0} = \infty$, $i = A, B$. We also assume that the cross-partial is not too negative: $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$, where $\kappa > 0$. This ensures that

[13] In the Appendix we also discuss the role of long-term contracts where asset owners can commit to future transaction prices. We ask whether such contracts can be used to structure more efficient ex-ante arrangements. However, we find that compared to joint asset ownership, a long-term contract cannot improve ex-ante efficiency. Hence our focus on optimal ownership structures.

the reaction functions of both partners are well-behaved.[14] Alice and Bob learn the actual inside prospect $\pi \in \{\pi_L, \pi_H\}$ at date 2.

Depending on the observed inside prospect $\pi \in \{\pi_L, \pi_H\}$ at date 2, Alice and Bob can decide to break their original partnership and match with alternative partners. Specifically we assume that Alice finds an alternative partner, called *Charles* (C), with probability $q > 0$. We assume symmetry so that Bob discovers an alternative partner, called *Dora* (D), with the same (but independent) probability q.[15] For simplicity we assume that both alternative partners, Charles and Dora, have zero wealth, and normalize their outside options to zero. The profit y of a successful alternative partnership has the distribution $\Omega_{out}(y)$. We denote the expected value by $\sigma = \int y d\Omega_{out}(y)$, which we refer to as the *outside prospect*.

At date 3 the partners engage in joint production; this can be either Alice and Bob (A, B), or Alice and Charles (A, C) and/or Bob and Dora (B, D). Joint production requires (i) the use of both of the partners' complementary assets, and (ii) their private efforts, which we denote e_i, $i = A, B, C, D$. A partner's disutility of effort is $c(e_i)$, with $c'(e_i) > 0$, $c''(e_i) > 0$, and $c(0) = c'(0) = 0$. Production either generates a joint profit at date 4 (success), or no profit at all (failure). The success probability is given by $\mu(e_i e_j)$, $i, j \in \{\{A, B\}, \{A, C\}, \{B, D\}\}$, which is increasing and concave in its argument $e_i e_j$, with $\mu(0) = 0$. Thus, the partners' efforts are complementary, and success requires that both partners apply strictly positive efforts (i.e., e_i, $e_j > 0$).

The realized profit y at date 4 can be divided between the two partners according to any sharing rule where Alice obtains αy and Bob receives βy, with $\alpha + \beta = 1$. Depending on the ownership structure this sharing rule can be implemented in different ways. Under joint asset ownership, we think of α and β as a division of ownership shares from the jointly owned venture. Under individual asset ownership there are no ownership shares, so the division of surplus comes from some transfer price.[16]

Ownership defines control rights over the productive assets. We assume that Alice and Bob initially have full rights of control over their respective assets. Alice and Bob can then choose

[14] A sufficient and intuitive assumption is that the specific investments r_A and r_B are (weak) strategic complements, so that $\partial^2 p(\cdot)/(\partial r_A \partial r_B) \geq 0$.

[15] Recall that Alice and Bob have complementary assets which both are needed for production. This excludes the possibility of Alice partnering with Dora, or Bob partnering with Charles.

[16] In principle it is possible to make α and β contingent on y. In the case of joint asset ownership, it is easy to verify that for any division of surplus with variable α and β, there exists an equivalent division of surplus with a constant α and β. W.l.o.g we can therefore focus on constant α's and β's. In the case of individual asset ownership, α and β depend on how transfer prices are specified, i.e., how they depend on the realization of y. To keep our notation as simple as possible we focus on the case of constant α's and β's. This is w.l.o.g since all that matters is the expected profit share at date 2.

to retain their control rights at date 0 (individual asset ownership); they can then simply wait until date 2 to see whether in fact they want to partner up. If they do, they negotiate a transfer price which determines their profit shares (α, β) at that time. Alternatively, the partners can agree at date 0 to share control rights over both assets (joint asset ownership). This requires that Alice and Bob negotiate the ownership shares α and β at date 0 (we discuss in the Online Appendix why we can limit ourselves to individual and joint asset ownership). Ownership matters because it affects the ability of a partner to leave: Under individual asset ownership, a partner with a superior outside option can always leave without the consent of the other. Under joint asset ownership, the two partners share control rights over both assets, so that leaving requires consent of the other partner.

The two initial partners Alice and Bob determine asset ownership at date 0. Bargaining can also occur at date 2, where it may involve two or more parties. Because of a potentially binding wealth constraint (in case each partner's initial wealth w is sufficiently low), we need a bargaining solution for games with non-transferable utilities. We adopt the bargaining protocol of Hart and Mas-Colell (1996), where in each round one member at the bargaining table is selected at random to make a proposal, and where there is a small probability that a partner whose proposal was rejected, is permanently eliminated from the bargaining.[17] This bargaining protocol generates the Maschler-Owen consistent NTU value, which is a generalization of the Shapley value for games with non-transferable utility (Maschler and Owen, 1992). For bilateral bargaining games, the Maschler-Owen consistent NTU value reduces to the Nash bargaining solution.[18]

We assume that the only members at the bargaining table are those who have the control rights to affect the decision. This means that under individual asset ownership, bargaining takes place between the two partners who want to engage in joint production. Under joint asset ownership, however, leaving requires the consent of the other partner. A new partner (Charles or Dora) therefore has to engage in trilateral bargaining with both of the original partners (Alice and Bob). In the Online Appendix we show that alternative bargaining protocols may generate different levels of utility, but they do not affect the basic logic of how partners make optimal asset ownership decisions.

[17]This is a multi-player generalization of the breakdown game by Binmore, Rubinstein, and Wolinsky (1986).

[18]For a more extensive discussion of this, see Hart (2004).

4 The Role of Asset Ownership

We first analyze how asset ownership affects the renegotiation outcome between Alice and Bob at date 2, and therefore their decision to stay together or to match with alternative partners. We then identify the optimal asset ownership that Alice and Bob agree on at date 0. To show that our key insights do not rely on specific investments, we deliberately shut down this part of the model until Section 5, and assume for now that the probability of a high inside prospect, p, is fixed.

4.1 Joint Production

In our model there is team production.[19] Alice and Bob choose their respective efforts e_A and e_B to maximize their expected utilities:[20]

$$\max_{e_A} U_A(\alpha;\pi) = \alpha\mu(e_A e_B)\pi - c(e_A) \qquad \max_{e_B} U_B(\beta;\pi) = \beta\mu(e_A e_B)\pi - c(e_B)$$

The analysis is analogous for joint production with a new partner (Charles or Dora) except that the inside prospect $\pi \in \{\pi_L, \pi_H\}$ is to be replaced by the outside prospect σ.

Figure 2 illustrates the utility-possibility frontier for different profit shares α and $\beta = 1-\alpha$. The frontier is backward bending because every partner relies on the productive effort of his co-partner. If one partner exerts no effort (which occurs when $\alpha \in \{0,1\}$), joint production never succeeds ($\mu(0) = 0$), and Alice and Bob both get a zero utility. We can also see from Figure 2 that the total surplus is maximized when each (symmetric) partner gets exactly half of the expected profit π ($\alpha = \beta = 1/2$); we formally prove this in the Online Appendix. However, each partner prefers to get more than half of the expected profit, i.e., the individually optimal profit shares for Alice and Bob satisfy $\alpha^{max} = \beta^{max} > 1/2$.

4.2 Symmetric Outside Options

We can now analyze how the allocation of control rights over the two assets, affects Alice's and Bob's decision at date 2 to stay together or to dissolve their partnership. For this we first consider the outcome in case they have identical outside options.

[19]Because of the binary nature of outcomes (success or failure), there is no possibility for budget breaking as in Holmström (1982).

[20]Note that wealth w will only indirectly affect the partners' effort incentives when it is used for transfer payments to change the profit shares α and β; we discuss this in more detail in Section 4.3.

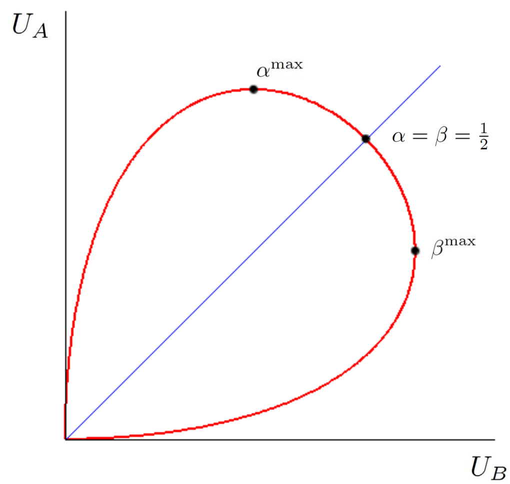

Figure 2: Utility-possibility Frontier for Joint Production

If neither Alice nor Bob found an alternative partner, which occurs with probability $(1-q)^2$, joint production is the only option. Because of symmetry, both partners share the profits equally, so that $\alpha^* = \beta^* = 1/2$. For this case we denote the expected utility of each partner by $U(\pi) \equiv U_A(\pi) = U_B(\pi)$.

Now suppose that Alice and Bob each found an alternative partner, which occurs with probability q^2. If Alice and Bob decide to stay together, they agree on $\alpha^* = \beta^* = 1/2$. The expected utility for each partners is then $U(\pi)$. Alternatively, Alice and Bob can decide to match with Charles and Dora respectively. They then bargain over the division of the expected profit σ from their new partnerships. Recall that the alternative partners, Charles and Dora, both have zero outside options. The same applies to Alice and Bob during the bargaining.[21] The Nash bargaining solution then implies that the profit shares for Alice and Bob in their new partnerships are given by $\widehat{\alpha} = \widehat{\beta} = 1/2$.[22] For this case we denote expected utility of each partner by $U(\sigma)$

It is straightforward to show that $U(\sigma) = U(\pi)$ when $\sigma = \pi \in \{\pi_L, \pi_H\}$. Thus, Alice and Bob stay together (joint production) with $\alpha^* = \beta^* = 1/2$ as long as $\pi \geq \sigma$. Otherwise they dissolve their partnership, and match with their alternative partners Charles and Dora.

[21] Intuitively, when negotiating a deal with Charles, Alice expects to close a deal, so she would not consider going back to Bob. Thus, Bob's outside option is zero when negotiating a deal with Dora at the same time, and vice versa. Technically, under the Hart and Mas-Colell bargaining protocol, there is an ε probability that the bargaining fails. Thus, with probability ε, Alice has the fall-back option of going back to Bob, and vice versa. The Hart and Mas-Colell bargaining protocol then assumes that $\varepsilon \to 0$, implying that Alice's and Bob's outside options converge to zero.

[22] Throughout the paper we use an asterisk (*) to indicate equilibrium profit shares under joint production (Alice-Bob match); a hat ($\widehat{\ }$) indicates the equilibrium profit shares in alternative matches (either Alice-Charles match, or Bob-Dora match).

And because leaving is mutually beneficial for $\sigma > \pi$, it is irrelevant whether they agreed on individual or joint asset ownership at date 0.

4.3 Asymmetric Outside Options

The most interesting scenario arises when only one partner found an alternative partner at date 2. This occurs with probability $q(1-q)$. We discuss the implications of individual and joint asset ownership separately.

4.3.1 Individual Asset Ownership

Suppose Alice and Bob agreed on individual asset ownership at date 0, and w.l.o.g. assume that only Alice found an alternative partner at date 2, Charles. To identify potential inefficiencies that may then arise, we first consider the case where Alice and Bob have no initial wealth ($w = 0$). We then relax this assumption and show how Alice and Bob can use their wealth to (partially) offset these inefficiencies.

Individual asset ownership allows Alice to unilaterally take her asset and form a new partnership with Charles without Bob's consent. The outside option of Alice when bargaining with her alternative partner Charles is to go back to Bob, and thus given by $U(\pi)$.[23] Let $\widehat{\alpha}_I$ denote the equilibrium profit share for Alice when partnering with Charles, where the subscript 'I' indicates individual asset ownership.[24] Alice's expected utility is then $U_A(\widehat{\alpha}_I; \sigma)$.

When Alice leaves Bob and matches with Charles, Bob's expected utility becomes $U_B = 0$. This is clearly smaller than his expected utility $U_B(\pi)$ under joint production with Alice. Thus, Alice imposes a *displacement externality* on Bob when displacing him with the alternative partner Charles. Leaving Bob is jointly inefficient when $U_A(\widehat{\alpha}_I; \sigma) < 2U(\pi)$.

Alice could also stay with Bob but use her better outside option (switching to Charles) to renegotiate a higher profit share. The outside option of Alice when renegotiating with Bob is given by $U(\sigma)$. Let α_I^* denote the equilibrium profit share for Alice when staying with Bob.[25] The renegotiation then leads to the expected utility $U_A(\alpha_I^*; \pi)$ for Alice, and $U_B(\beta_I^*; \pi)$ for Bob, with $\beta_I^* = 1 - \alpha_I^*$. Relative to the equal division of profits with $\alpha = \beta = 1/2$, this outcome

[23]According to the Hart and Mas-Colell bargaining protocol, this outside option would only be realized if the bargaining between Alice and Charles breaks down, so that Alice loses Charles as a potential trading partner. In this case, both Alice and Bob would have zero outside options, so that they split the equity in half.

[24]More formally, $\widehat{\alpha}_I$ maximizes the Nash product $[U_A(\widehat{\alpha}_I; \sigma) - U(\pi)]^{1/2}[U_C(1 - \widehat{\alpha}_I; \sigma)]^{1/2}$. It is easy to see that for any $U(\pi) > 0$ we have $\widehat{\alpha}_I \in (1/2, 1)$.

[25]Using Nash bargaining, α_I^* maximizes $[U_A(\alpha_I^*; \pi) - U(\sigma)]^{1/2}[U_B(1-\alpha_I^*; \pi)]^{1/2}$. Moreover, note that $U(\sigma) > 0$ implies $\alpha_I^* \in (1/2, 1)$.

is more favorable to Alice, and less favorable to Bob. Most importantly, it is jointly inefficient since joint surplus is maximized at $\alpha = \beta = 1/2$.

Whether the partner with the outside option stays or leaves the initial partnership depends on the inside project $\pi \in \{\pi_L, \pi_H\}$ that the two partners observe at date 2. In the Online Appendix we derive a threshold of the inside prospect, $\widehat{\pi}_I(\sigma) = \sigma$, so that asymmetric outside options under individual asset ownership with zero wealth lead to displacement when $\pi < \widehat{\pi}_I(\sigma)$, and unequal profit shares when $\pi \geq \widehat{\pi}_I(\sigma)$.

If Alice and Bob have some wealth $w > 0$, they could make transfer payments to (partially) offset these inefficiencies. With unlimited wealth the ex-post inefficiencies can be completely eliminated (Alice then stays with $\alpha = \beta = 1/2$). In the Online Appendix we characterize the minimum amount of wealth, denoted $\overline{w}_I$, that is required to fully eliminate the inefficiencies. We also characterize $\underline{w}_I \geq 0$ as the lower bound, below which wealth cannot change the renegotiation outcome (Alice still prefers to partner with Charles).

Lemma 1 *Consider individual asset ownership and suppose that the two original partners have asymmetric outside options. Then, there exists a threshold $\widehat{\pi}_I(\sigma, w)$ such that the partner with the better outside option leaves if $\pi < \widehat{\pi}_I(\sigma, w)$. Otherwise, if $\pi \geq \widehat{\pi}_I(\sigma, w)$, he stays but renegotiates his share on the expected joint profit π. The threshold $\widehat{\pi}_I(\sigma, w)$ is decreasing in wealth w for $\underline{w}_I \leq w < \overline{w}_I$.*

Joint production between Alice and Bob is the outcome under individual asset ownership with asymmetric outside options whenever the inside prospect π is sufficiently high ($\pi \geq \widehat{\pi}_I(\sigma, w)$). The partner with the better outside option then renegotiates the division of surplus, which is optimal from a selfish perspective but compromises the efficiency of joint production (as long as the partners do not have sufficient wealth for transfers to settle on an equal split of profits). Displacement, on the other hand, occurs whenever the prospect of the original partnership is sufficiently low ($\pi < \widehat{\pi}_I(\sigma, w)$). This imposes a displacement externality on the partner without outside option.

The effect of more initial wealth w is to allow the partner without outside option to offer a larger transfer payment. This makes staying (with renegotiation) more attractive for the partner with the outside option. As a consequence the region where the original partners stay together is larger, i.e., the critical value $\widehat{\pi}_I$ becomes smaller.

4.3.2 Joint Asset Ownership

Now consider joint asset ownership, and w.l.o.g. assume again that only Alice found an alternative partner, Charles, at date 2.

Suppose Alice wants to leave Bob. Without wealth, Alice can only buy out her asset by offering Bob a share on the future return σ from her new partnership with Charles. Productive effort is then only applied by Alice and Charles, so that Bob is a shareholder who does not add any value. We define $\widehat{\alpha}_J$ and $\widehat{\beta}_J$ as the equilibrium shares on the return σ for Alice and Bob, respectively. The equilibrium share for Charles is denoted by $\widehat{\gamma}_J$.[26] For this scenario we denote the expected utility for Alice as $U_A(\widehat{\alpha}_J;\sigma)$, and for Bob as $U_B(\widehat{\beta}_J;\sigma)$. This buyout arrangement impairs effort incentives, and thus lowers the expected payoff from Alice's partnership with Charles.

However, the profit share $\widehat{\beta}_J$ offered to Bob may not suffice to buy his consent, so that Alice is forced to stay despite leaving being jointly efficient, which is the case when $U(\sigma) > 2U(\pi)$. Bob then imposes a retention externality on Alice.

Which of the two inefficiencies – inefficient buyout or retention – arises in equilibrium, depends again on the inside project $\pi \in \{\pi_L, \pi_H\}$ that Alice and Bob observe at date 2. In the Online Appendix we characterize the threshold $\widehat{\pi}_J(\sigma)$, so that asymmetric outside options under joint asset ownership with zero wealth lead to retention when $\pi \geq \widehat{\pi}_J(\sigma)$, and inefficient buyout when $\pi < \widehat{\pi}_J(\sigma)$.

When Alice and Bob have some wealth $w > 0$, they can make side-payments to mitigate these inefficiencies. In the Online Appendix we characterize the minimum amount of wealth, denoted $\overline{w}_J$, that is necessary to eliminate all ex-post inefficiencies under joint asset ownership (Alice can buy out her asset without offering Bob a profit share). Likewise we characterize the lower bound of wealth, $\underline{w}_J \geq 0$, below which wealth cannot affect the renegotiation outcome (the transfer is not enough for Bob to let Alice go).

Lemma 2 *Consider joint asset ownership and suppose that the two original partners have asymmetric outside options. Then, there exists a threshold $\widehat{\pi}_J(\sigma, w)$, such that the partner with the better outside option leaves with consent if $\pi < \widehat{\pi}_J(\sigma, w)$. Otherwise, if $\pi \geq \widehat{\pi}_J(\sigma, w)$, he stays with $\alpha^* = \beta^* = 1/2$. The threshold $\widehat{\pi}_J(\sigma, w)$ is increasing in wealth w for $\underline{w}_J \leq w < \overline{w}_J$.*

Joint production between Alice and Bob is the equilibrium outcome as long as the inside prospect π is sufficiently high ($\pi \geq \widehat{\pi}_J(\sigma, w)$). They then split everything in half ($\alpha^* = \beta^* = 1/2$), so that total surplus is maximized. In contrast, Alice and Bob agree to break up whenever the alternative partnership is attractive enough ($\pi < \widehat{\pi}_J(\sigma, w)$). Unless the partner with the better outside option, say Alice, has sufficient wealth ($w \geq \overline{w}_J$), she needs to offer Bob a stake in the new partnership with Charles in exchange for regaining control rights over her asset. The

[26] We provide a complete characterization of $\widehat{\alpha}_J$, $\widehat{\beta}_J$, and $\widehat{\gamma}_J$ in the Online Appendix, using the Maschler-Owen consistent NTU value.

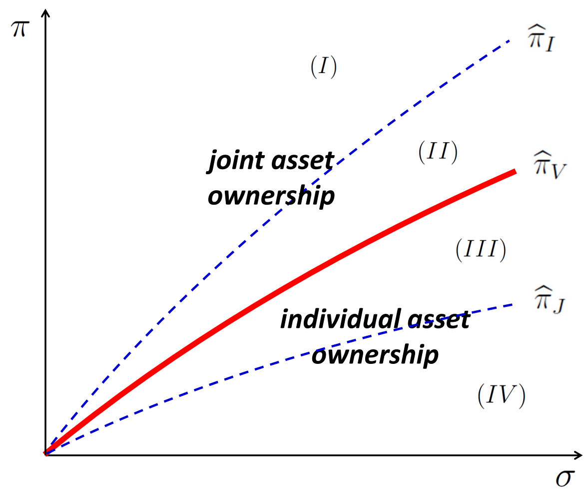

Figure 3: Asset Ownership and Joint Efficiency

effect of more wealth w is to enable Alice to make larger payments to Bob, thereby allowing her to retain more of the equity of the new partnership with Charles. This makes the buyout option more attractive, and leaving occurs for a larger range of parameters, i.e., the critical value $\widehat{\pi}_J$ is increasing in w.

4.4 Optimal Asset Ownership

With symmetric outside options it is irrelevant whether Alice and Bob agreed on individual or joint asset ownership; the outcome is always jointly efficient. Only for asymmetric outside options and binding wealth constraints the allocation of control rights matters.

Proposition 1 *Suppose $w < \min\{\overline{w}_I, \overline{w}_J\}$. Then, there exists a threshold $\widehat{\pi}_V(\sigma)$, such that the contract choice for the two partners at date 0 is as follows:*

(i) *For $\pi_L, \pi_H < \widehat{\pi}_V(\sigma)$, they choose individual asset ownership.*

(ii) *For $\pi_L, \pi_H \geq \widehat{\pi}_V(\sigma)$, they choose joint asset ownership.*

(iii) *For $\pi_L < \widehat{\pi}_V(\sigma) < \pi_H$, there exists a threshold $\widehat{p} \in (0, 1)$, such that both partners choose joint asset ownership at date 0 whenever $p \geq \widehat{p}$; otherwise they choose individual asset ownership.*

The threshold $\widehat{\pi}_V(\sigma)$ satisfies $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_V(\sigma) < \widehat{\pi}_I(\sigma, w)$, and is increasing in σ.

For now let us focus on the scenarios (i) and (ii) to explain the rationale behind Proposition 1. For this we refer to Figure 3, which presumes that Alice and Bob have asymmetric outside options at date 2. Dissolving their partnership is then jointly efficient if the inside prospect is sufficiently low ($\pi < \widehat{\pi}_V(\sigma)$); otherwise joint production with an equal split of profits maximizes joint surplus.

Consider regions (I) and (II). In these two regions the inside prospect π is sufficiently high, so that Alice's and Bob's joint utility is maximized when they remain together. Under individual asset ownership there is a displacement externality: In region (II) the outside prospect σ is sufficiently attractive, so that the partner with outside option simply leaves without renegotiation. In region (I), the partner with outside option merely uses his opportunity to switch as a bargaining chip. Both of these outcomes are ex-post inefficient from a joint perspective. These inefficiencies can be avoided with joint asset ownership, where Alice and Bob always remain together without renegotiation.

In regions (III) and (IV), the inside prospect is weak relative to the outside prospect, so that dissolving the partnership in case of asymmetric outside options would maximize Alice's and Bob's joint surplus. In region (IV) the partner with outside option has to buy himself free under joint asset ownership, which compromises effort incentives in his new partnership. However, in region (III) the outside prospect is not high enough to warrant a buyout. As a consequence the partner without outside option inefficiently retains the other. Obviously, these retention inefficiencies can be avoided with individual asset ownership.

Now consider the most interesting scenario (iii) from Proposition 1. Alice and Bob then choose joint asset ownership if the inside prospect π is likely to be high ($p \geq \widehat{p}$), because preserving the partnership is likely to be valuable. Otherwise they choose individual asset ownership in order to retain the flexibility to dissolve a likely inefficient partnership ($p < \widehat{p}$). Thus, the threshold $\widehat{p}$ balances (i) the risk of preserving inefficient partnerships (joint ownership with $\pi = \pi_L$), and (ii) the risk of compromising otherwise efficient partnerships (individual ownership with $\pi = \pi_H$). Overall we note that for this scenario the ex-ante optimal allocation of control rights can lead to ex-post inefficiencies, namely a displacement inefficiency (individual ownership), associated with regions (I) and (II) in Figure 3, and a retention externality (joint ownership), associated with regions (III) and (IV).

If the internal learning process was based on verifiable signals so that an ex-ante contract can distinguish between $\pi = \pi_L$ and $\pi = \pi_H$, then Alice and Bob could write a contingent contract which stipulates individual asset ownership at date 2 whenever $\pi = \pi_L < \widehat{\pi}_V(\sigma)$, and joint asset ownership whenever $\pi = \pi_H \geq \widehat{\pi}_V(\sigma)$. For the remainder of this paper we assume that π is non-verifiable, which seems very plausible within the present context, given that learning about the inside prospect is specific to the collaboration of the two partners. Moreover, we focus on the most interesting scenario where $\pi_L < \widehat{\pi}_V(\sigma) < \pi_H$. This implies that the ex-ante decision over asset ownership involves a trade-off between the flexibility of individual asset ownership versus the commitment value of joint asset ownership.

5 Relation-specific Investments

We now augment our model and allow Alice and Bob to make relation-specific investments at date 1.

Note that both partners are symmetric at date 1, so that in equilibrium they choose the same level of specific investment. We define $r_I^*(w)$ as the equilibrium relation-specific investment of a partner under individual asset ownership, and $r_J^*(w)$ as the equilibrium investment under joint asset ownership. We provide a complete characterization of the partners' expected utilities and the equilibrium investment levels, $r_I^*(w)$ and $r_J^*(w)$, in the Online Appendix (see Proof of Proposition 2).

For now let us assume that Alice and Bob use their entire wealth to mitigate ex-post inefficiencies. For this the next proposition compares the specific investments under individual asset ownership ($r_I^*(w)$) and joint asset ownership ($r_J^*(w)$) for different wealth levels.

Proposition 2 *For $w < \max\{\overline{w}_I, \overline{w}_J\}$, joint asset ownership provides greater incentives for relation-specific investments, i.e., $r_J^*(w) > r_I^*(w)$. Moreover,*

(i) *$r_J^*(w)$ is decreasing in the partners' wealth w for $\underline{w}_J \leq w < \overline{w}_J$, and*

(ii) *$r_I^*(w)$ is increasing in the partners' wealth w for $\underline{w}_I \leq w < \overline{w}_I$.*

For $w \geq \max\{\overline{w}_I, \overline{w}_J\}$, relation-specific investments are identical and constant under individual and joint asset ownership, i.e., $r_I^*(w) = r_J^*(w)$.

Figure 4 illustrates the insights from Proposition 2 (allowing for all renegotiation scenarios: $\underline{w}_i = 0$ and $\underline{w}_i > 0$, $i = I, J$). To explain the key intuition, let us first focus on the case with zero wealth ($w = 0$), so that Alice and Bob cannot make any ex-post transfers. With joint asset ownership, the partner combination is always efficient when the inside prospect is high, but leads to inefficient retention when the inside prospect is low. The latter inefficiency of joint asset ownership widens the difference in utilities between a low and a high inside prospect. With individual asset ownership, the partner combination is always efficient when the inside

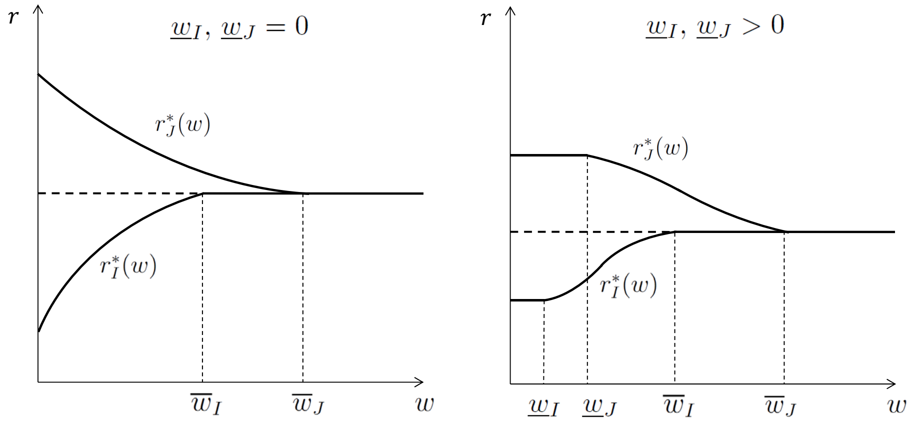

Figure 4: Wealth and Relation-specific Investments

prospect is low, but causes displacement problems when the inside prospect is high. The latter inefficiency of individual asset ownership narrows the difference in utilities between a low and a high inside prospect. We therefore find that joint asset ownership provides stronger incentives for specific investments ($r_J^*(0) > r_I^*(0)$), precisely because the inefficiency then arises when the partners have failed to develop a strong internal relationship .

The next question is what happens to specific investments when Alice and Bob have some initial wealth $w > 0$? Under individual asset ownership, wealth allows them to smooth out ex-post inefficiencies in the good state $\pi = \pi_H$. This improves the marginal benefit of specific investments, so that $r_I^*(w)$ is increasing in w. Under joint asset ownership, wealth helps Alice and Bob to correct ex-post inefficiencies in the bad state $\pi = \pi_L$. This makes the difference between the bad and the good state relatively smaller, and therefore compromises Alice's and Bob's incentives to make relation-specific investments. Thus, $r_J^*(w)$ decreases in w, while $r_I^*(w)$ increases.

Alice and Bob can eliminate all ex-post inefficiencies in case of asymmetric outside options when they have sufficient ($w \geq \max\{\overline{w}_I, \overline{w}_J\}$). That is, with enough wealth they can always dissolve their partnership in the bad state (π_L), so that $V(\pi_L) = U(\sigma)$; and they can always agree on staying together with an equal split of profits in the good state (π_H), so that $V(\pi_H) = 2U(\pi)$. The allocation of control rights is then irrelevant, and the marginal incentives for specific investments are the same ($r_I^*(w) = r_J^*(w)$).

6 Specific Investments and Optimal Asset Ownership

We can now complete our model and identify the optimal ownership structure for Alice and Bob, accounting for their specific investments and potential ex-post transfers.

We first characterize Alice's and Bob's expected utilities for different levels of wealth, assuming again that they use their entire wealth to mitigate ex-post inefficiencies.[27]

Lemma 3 *Under individual asset ownership, the expected utility of a partner at date* 0*, denoted by* $EU_I(w)$*, has three distinct segments:*

(i) *For* $w < \underline{w}_I$*,* $EU_I(w)$ *is constant in* w*.*

(ii) *For* $\underline{w}_I \leq w < \overline{w}_I$*,* $EU_I(w)$ *is strictly increasing in* w*.*

(iii) *For* $w \geq \overline{w}_I$*,* $EU_I(w)$ *is constant in* w*.*

The previous sections identified two distinct facets of wealth. On the one hand, having wealth allows Alice and Bob to mitigate potential inefficiencies arising from asymmetric outside options; and doing so is always optimal ex-post. On the other hand, having wealth affects their incentives for relation-specific investments. Under individual asset ownership the ex-post *efficiency effect* of wealth and the *incentive effect* of wealth both go in the same direction: More wealth improves the renegotiation outcome at date 2; it also improves ex-post incentives for specific investments at date 1 because the inefficiencies are associated with a high inside prospect. The expected utility $EU_I(w)$ is therefore increasing in wealth w in the range $w \in [\underline{w}_I, \overline{w}_I)$, and constant everywhere else.

We now turn to joint asset ownership. For the next lemma we define w_J^* as the wealth level which maximizes the expected utility of a partner under joint ownership at date 0.

Lemma 4 *Under joint asset ownership, the expected utility of a partner at date* 0*, denoted by* $EU_J(w)$*, has the following properties:*

(i) *For* $w < \underline{w}_J$*,* $EU_J(w)$ *is constant in* w*.*

(ii) *For* $\underline{w}_J \leq w < \overline{w}_J$*, there exists a threshold* $\widehat{\pi}_H$ *such that* $w_J^* = \underline{w}_J$ *for all* $\pi_H \geq \widehat{\pi}_H$*, and* $w_J^* \in (\underline{w}_J, \overline{w}_J)$ *for all* $\pi_H < \widehat{\pi}_H$*.*

[27] The expected utility is obviously increasing in wealth itself, so we focus on the expected utility from the productive activities, net of initial wealth. This expected utility still depends on wealth, since wealth affects both incentives and ex-post payoffs (case of asymmetric outside options).

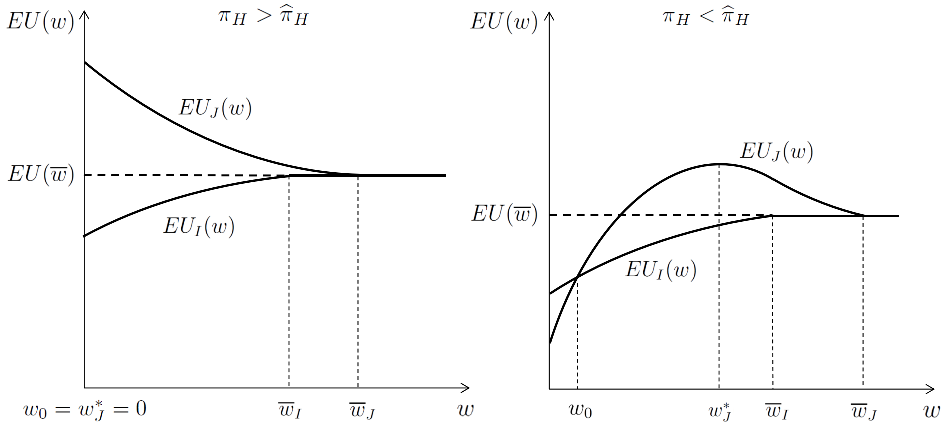

Figure 5: Wealth and Expected Utilities under Individual and Joint Asset Ownership

If $w_J^* = \underline{w}_J$, *then* $EU_J(w)$ *is strictly decreasing in* w *for* $w \in [\underline{w}_J, \overline{w}_J)$.

If $w_J^* > \underline{w}_J$, *then* $EU_J(w)$ *is strictly increasing in* w *for* $w \in [\underline{w}_J, w_J^*)$, *and strictly decreasing in* w *for* $w \in (w_J^*, \overline{w}_J)$.

(iii) *For* $w \geq \overline{w}_J$, $EU_J(w)$ *is constant in* w.

Lemma 4 shows that a partner's expected utility under joint asset ownership is not necessarily monotone in wealth. This is because wealth has two opposite effects: It allows Alice and Bob in case of asymmetric outside options to improve their ex-post payoffs in the bad state $\pi = \pi_L$. However, this concurrently compromises Alice's and Bob's incentives to invest in their relationship (see Proposition 2). Which effect dominates then depends on the importance of relation-specific investments, as reflected by the parameter π_H. For sufficiently high values of π_H ($\pi_H \geq \widehat{\pi}_H$), the incentive effect always dominates. In this case the expected utility $EU_J(w)$ is decreasing in w, and has its maximum at zero wealth.[28] For lower values of π_H ($\pi_H < \widehat{\pi}_H$), the incentive effect does not always dominate. In the Online Appendix we show that the expected utility $EU_J(w)$ then first increases in wealth, and then decreases.

We can now derive a condition so that Alice and Bob prefer joint to individual asset ownership at date 0. For parsimony we define $\overline{w} \equiv \max\{\overline{w}_I, \overline{w}_J\}$.

Proposition 3 *There always exists a critical wealth level* w_0, *with* $w_0 \in [0, w_J^*]$, *such that the partners strictly prefer joint asset ownership for all* $w \in (w_0, \overline{w})$.

Figure 5 compares the expected utility levels under individual versus joint asset ownership. If Alice and Bob have sufficient wealth ($w \geq \overline{w}$), they can eliminate all ex-post inefficiencies

[28]If $\underline{w}_J > 0$, there is a range $[0, \underline{w}_J]$ where $EU_J(w)$ is maximized.

in case of asymmetric outside options. The specific ownership structure is then irrelevant (i.e., $EU_I(w) = EU_J(w)$ for $w \geq \overline{w}$). For $w < \overline{w}$, however, there always exists a region where joint asset ownership is preferred to individual asset ownership. This region extends all the way down to w_0. In some cases we have $w_0 = 0$, so that joint asset ownership is optimal for all levels of wealth; see the left graph of Figure 5 where $\pi_H \leq \widehat{\pi}_H$. In other cases we have $w_0 > 0$, so that joint asset ownership is only optimal for intermediate levels of wealth; see the right graph of Figure 5 where $\pi_H > \widehat{\pi}_H$. All this implies that the two partners, Alice and Bob, only choose individual asset ownership at date 0 when (i) relation-specific investments are not very important for their partnership ($\pi_H \leq \widehat{\pi}_H$), and (ii) they have little or no initial wealth ($w \leq w_0$). Otherwise they always have a (weak) preference for joint asset ownership.

We can see from Figure 5 that the partners' expected utilities depend on how much wealth they have available for ex-post transfers. An interesting question is what wealthy partners would do if they can commit to limiting the amounts that can be used for ex-post transfer payments? We get the following corollary, which immediately follows from the above, and can be seen directly off Figure 5.

Corollary 1 *If wealthy partners can commit to limiting the wealth available for ex-post transfer payments, then they always choose joint asset ownership and commit to being wealth constrained at $w = w_J^*$.*

The maximum of the expected utilities, $EU^{\max} = \max\{EU_I(w), EU_J(w)\}$, is always reached at $EU_J(w_J^*)$. This implies that the combination of joint asset ownership and a wealth constraint at w_J^* achieves the best trade-off between ex-ante incentives for specific investments and ex-post efficiency. Interestingly, in the case of $\pi_H \geq \widehat{\pi}_H$, we even have $w_J^* = 0$. The optimal arrangement for wealthy partners is then joint asset ownership with the commitment to a zero wealth constraint ex-post.

7 Asymmetric Partners

In our base model we focused on two symmetric partners. Naturally one may ask whether our key trade-off between displacement and retention externalities remains intact when allowing for asymmetric partners. We focus on two sources of asymmetry: asymmetric wealth and asymmetric outside options.

Suppose the two partners have different initial wealth levels. For example, one of the partners may be a wealth-constrained entrepreneur, the other an established corporation with large

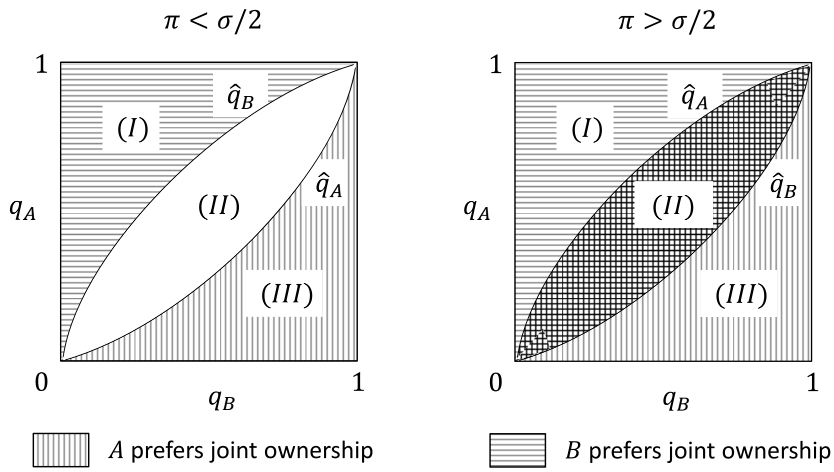

Figure 6: Preferences with Ex-ante Asymmetric Outside Options

cash reserves. For simplicity suppose that Alice is fully wealth constrained but Bob faces no such constraints. Consider individual asset ownership with a high inside prospect (π_H). If only Alice finds an outside partner, she may want to leave. This causes a displacement externality. Now if Bob has wealth, he can make a transfer payment that convinces Alice to stay, making both parties better off. Unfortunately this solution only works in one of the two asymmetric scenarios. If only Bob finds an outside partner, he may want to leave. Alice does not have the wealth to retain Bob, and thus the displacement externality arises again in equilibrium. A similar argument applies to joint asset ownership with a low inside prospect (π_L). If only Bob finds an outside partner, he normally is affected by the retention externality. If Bob has wealth, he can make a transfer payment to buy out Alice. However, if only Alice finds an outside partner, she cannot buy herself free, and the retention externality arises again in equilibrium. Overall we see that with asymmetrically wealthy partners the same inefficiencies occur, only less frequently. All that is needed for our key model insights is that at least one of the partners has insufficient wealth to completely eliminate potential displacement and retention externalities.

It remains to discuss how asymmetric expectations about their outside options affect the partners' assets ownership decisions. For this we use a simplified version of our model without specific investments, where Alice and Bob each get a utility $\pi/2$ when staying together, and $\sigma/2$ when matching with alternative partners. We also assume that Alice will find an alternative partner, Charles, with probability q_A, while Bob will find Dora with probability $q_B(\neq q_A)$.

In the Online Appendix we show that Alice prefers joint asset ownership if

$$q_A < \widehat{q}_A = \left[\left(\frac{1-q_B}{q_B}\right)\left(\frac{\sigma-\pi}{\pi}\right)+1\right]^{-1}.$$

Likewise, Bob favors joint ownership if $q_B < \widehat{q}_B$, where $\widehat{q}_B$ is symmetric to $\widehat{q}_A$. The threshold $\widehat{q}_A$ ($\widehat{q}_B$) is increasing and convex in q_B (q_A) when $\pi < \sigma/2$. When $\pi > \sigma/2$, it is increasing and concave in q_B (q_A)

Figure 6 illustrates Alice's and Bob's preferences for joint asset ownership, for different values of q_A (y-axis) and q_B (x-axis). Consider first the left graph where $\pi < \sigma/2$, i.e., where leaving is efficient from a joint perspective. In region (III), Alice is sufficiently unlikely to find an alternative partner, and therefore prefers the protection of joint asset ownership ($q_A < \widehat{q}_A$). Likewise, Bob only prefers joint asset ownership in region (I) ($q_B < \widehat{q}_B$). We can see that for $\pi < \sigma/2$ the two partners never agree on sharing control rights. Thus, individual asset ownership is the equilibrium outcome for all $q_A, q_B \in (0, 1)$.

The right graph illustrates Alice's and Bob's preferences when $\pi > \sigma/2$, i.e., when staying together maximizes their joint surplus. Again, each partner prefers joint asset ownership only when he is sufficiently unlikely to find an alternative match. For Alice this happens in regions (II) and (III), for Bob in regions (I) and (II). In region (II) Alice and Bob both prefer to share control over their assets. Joint asset ownership therefore requires that the two partners are not too dissimilar in terms of their outside prospects.

8 Conclusion

This paper develops a dynamic theory of optimal firm boundaries based on partner uncertainty. The model identifies a fundamental trade-off between two ex-post inefficiencies. Under non-integration (i.e., individual asset ownership) there can be a *displacement externality*, where a partner may leave even though the benefit is worth less than the loss to the displaced partner. Under integration (i.e., joint asset ownership) there can be a *retention externality*, where one partner may hold on to the other, even though the benefit to the departing partner would exceed the loss to the remaining partner. Moreover, we show that wealth has two distinct effects. Ex-post, wealth mitigates the displacement and retention externalities. Ex-ante, however, wealth reduces incentives for specific investments. We also find that wealthy owners always want to commit ex-ante to limiting ex-post transfer payments.

Our model generates some new empirical predictions about the dynamics of firm boundaries. Most of the prior theoretical work focuses on comparative statics, and consequently most of the empirical work emphasizes cross-sectional determinants of the integration decision. Our theory suggests an empirical research agenda about the time-series properties of integration. For a given level of asset specificity, our model predicts that partner switching is more common under individual than under joint asset ownership. It seems highly intuitive that switching to outside buyers or suppliers is rarer in a vertically integrated setting. The interesting point is that this simple prediction cannot be obtained from theories with ex-post efficiency: in these models partners always switch exactly at the efficient time, irrespective of asset ownership. Our model also predicts that non-integration is more common in environments with high partner uncertainty. Prior empirical work typically focused on general measures of uncertainty, often with mixed results (see Lafontaine and Slade, 2007). We contend that these measures fail to distinguish between uncertainty about production and demand, versus uncertainty about partner choices.[29]

Our analysis suggests avenues for further theoretical work. We focused on team incentives and wealth constraints as source of ex-post inefficiencies; but there may be other sources of inefficiencies, such as asymmetric information (Aghion et al., 2012). Future research may therefore examine how alternative ex-post inefficiencies affect the dynamic properties of firm boundaries. In our model the arrival of superior partners is exogenous. Another interesting extension would be to consider the strategies that firms choose to identify alternative partners. Finally, we chose the simplest possible dynamic specification where partners have at most one opportunity to switch. A worthwhile future research agenda is to extend the model to an infinite horizon. This would allow for a more comprehensive analysis of how asset ownership affects the timing and frequency of partner changes. Overall we believe that looking at the dynamics of asset ownership over time constitutes a new and promising direction for future research.

[29]Recent work by Fresard, Hoberg and Phillips (2013) provides evidence that indirectly supports our perspective. Using a text-based analysis to measure vertical relationships, they find a positive relationship between firm age and vertical integration.

References

Aghion, Philippe, and Patrick Bolton (1987). Contracts as a Barrier to Entry. *American Economic Review*, 77(3), 388-401.

Aghion, Philippe, and Patrick Bolton (1992). An Incomplete Contracts Approach to Financial Contracting. *Review of Economic Studies*, 59(3), 473-494.

Aghion, Philippe, Drew Fudenberg, Richard Holden, Takashi Kunimoto, and Olivier Tercieux (2012). Subgame Perfect Implementation Under Information Perturbations and the Hold-Up Problem. *Quarterly Journal of Economics*, 127 (4), 1843-1881.

Bajari, Patrick, and Steven Tadelis (2001). Incentives versus Transaction Costs: A Theory of Procurement Contracts. *RAND Journal of Economics*, 32(3), 387-407.

Binmore, Ken, Ariel Rubinstein, and Asher Wolinsky (1986). The Nash Bargaining Solution in Economic Modelling. *RAND Journal of Economics*, 17(2), 176-188.

Blonski, Matthias, and Giancarlo Spagnolo (2003). Relational Contracts and Property Rights. Working Paper.

Bolton, Patrick, and Michael D. Whinston (1993). Incomplete Contracts, Vertical Integration, and Supply Assurance. *Review of Economic Studies*, 60(1), 121-148.

Brien, Michael J., Lee A. Lillard, and Steven Stern (2006). Cohabitation, Marriage, and Divorce in a Model of Match Quality. *International Economic Review*, 47(2), 451-494.

Cai, Hongbin (2003). A Theory of Joint Asset Ownership. *RAND Journal of Economics*, 34(1), 63-77.

Casas-Arce, Pablo, and Thomas Kittsteiner (2011). Opportunism and Incomplete Contracts. Working Paper.

Crampton, Peter, Robert Gibbons, and Paul Klemperer (1987). Dissolving a Partnership Efficiently. *Econometrica*, 55(3), 615-632.

de Fontenay, Catherine, Joshua S. Gans, and Vivienne Groves (2010). Exclusivity, Competition and the Irrelevance of Internal Investment. *International Journal of Industrial Organization*, 28, 336-340.

de Frutos, Maria-Angeles, and Thomas Kittsteiner (2008). Efficient Partnership Dissolution under Buy-sell Clauses. *RAND Journal of Economics*, 39(1), 184-198.

Franco, April, Matthew Mitchell, and Galina Vereshchagina (2011). Incentives and the Structure of Teams. *Journal of Economic Theory*, 146(6), 2307-2332.

Fresard, Laurent, Gerard Hoberg, and Gordon Phillips (2013). The Incentives for Vertical Mergers and Vertical Integration. Working Paper.

Friedberg, Leora (1998). Did Unilateral Divorce Raise Divorce Rate? Evidence from Panel Data. *American Economic Review*, 88(3), 608-627.

Gans, Joshua (2005). Markets for Ownership. *RAND Journal of Economics*, 36(2), 433-455.

Gans, Joshua S. and Scott Stern (2003). The Product Market and the Market for "Ideas": Commercialization Strategies for Technology Entrepreneurs. *Research Policy* 32(2), 333-350.

Gibbons, Robert (2005). Four Formal(izable) Theories of the Firm? *Journal of Economic Behavior & Organization*, 58, 200-245.

Grossman, Sanford J., and Oliver D. Hart (1986). The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration. *Journal of Political Economy*, 94(4), 691-719.

Grossman, Sanford J., and Oliver D. Hart (1987). Vertical Integration and the Distribution of Property Rights, in: Razin, Assaf and Efraim Sadka (Eds.), Economic Policy in Theory and Practice, Macmillan Press, 504-546.

Halonen, Maija (2002). Reputation and the Allocation of Ownership. *Economic Journal*, 112(481), 539-558.

Hart, Sergiu (2004). A Comparison of Non-transferable Utility Values. *Theory and Decision*, 56(1), 35-46.

Hart, Sergiu, and Andreu Mas-Colell (1996). Bargaining and Value. *Econometrica*, 64(2), 357-380.

Hart, Oliver (1995). Firms, Contracts and Financial Structure. Clarendon Press. Oxford.

Hart, Oliver (2009). Hold-Up, Asset Ownership, and Reference Points. *Quarterly Journal of Economics*, 124(1), 267-300.

Hart, Oliver, and Bengt Holmstrom (2010). A Theory of Firm Scope. *Quarterly Journal of Economics*, 125 (2), 483-513.

Hart, Oliver, and John Moore (1990). Property Rights and the Nature of the Firm. *Journal of Political Economy*, 98(6), 1119-1158.

Hellmann, Thomas F. and Enrico Perotti (2011). The Circulation of Ideas in Firms and Markets. *Management Science*, 57(10), 1813-1826.

Hellmann, Thomas F., and Veikko Thiele (2015). Contracting among Founders. *Journal of Law, Economics, & Organization*, 31(3), 629-661.

Holmström, Bengt (1982). Moral Hazard In Teams. *Bell Journal Of Economics*, 13(2), 324-340.

Jing, Ran, and Ralph Winter (2014). Exclusionary Contracts. *Journal of Law, Economics, & Organization* 30(4), 833-867.

Lafontaine, Francine, and Margaret Slade (2007). Vertical Integration and Firm Boundaries: The Evidence. *Journal of Economic Literature*, 45(3), 629-685.

Maschler, Michael, and Guillermo Owen (1992). The Consistent Shapley Value for Games without Side Payments. In: Reinhard Selten (Ed.), Rational Interaction, New York: Springer-Verlag, 5-12.

Matouschek, Niko (2004). Ex Post Inefficiencies in a Property Rights Theory of the Firm. *Journal of Law, Economics, & Organization*, 20(1), 125-147.

Matouschek, Niko, and Imran Rasul (2008). The Economics of the Marriage Contract: Theories and Evidence. *Journal of Law and Economics*, 51(1), 59-110.

McAfee, R. Preston (1992). Amicable Divorce: Dissolving a Partnership with Simple Mechanisms. *Journal of Economic Theory*, 56(2), 266-293.

Nöldeke, Georg, and Klaus M. Schmidt (1998). Sequential Investments and Options to Own. *RAND Journal of Economics*, 29(4), 633-653.

Peters, Elizabeth H. (1986). Marriage and Divorce: Informational Constraints and Private Contracting. *American Economic Review*, 76(3), 437-454.

Peters, Michael (2007). The Pre-marital Investment Game. *Journal of Economic Theory*, 137(1), 186-213.

Peters, Michael, and Aloysius Siow (2002). Competing Premarital Investments. *Journal of Political Economy*, 110(3), 592-608.

Prat, Andrea (2002). Should a Team be Homogeneous? *European Economic Review*, 46(7),1187-1207.

Segal, Ilya R., and Michael D. Whinston (2000). Exclusive Contracts and Protection of Investments. *RAND Journal of Economics*, 31(4), 603-633.

Segal, Ilya R., and Michael D. Whinston (2012). Property Rights. In: Robert Gibbons and John Roberts (Eds.), Handbook of Organizational Economics.

Tadelis, Steven (2002). Complexity, Flexibility, and the Make-or-Buy Decision. *American Economic Review*, 92(2), 433-37.

Whinston, Michael D. (2003). On the Transaction Costs Determinants of Vertical Integration. *Journal of Law, Economics, & Organization*, 19(1), 1-23.

Wickelgren, Abraham L. (2009). Why Divorce Laws Matter: Incentives for Noncontractible Marital Investments under Unilateral and Consent Divorce. *Journal of Law, Economics, & Organization*, 25(1), 80-106.

Williamson, Oliver E. (1975). Markets and Hierarchies: Analysis and Antitrust Implications. The Free Press, New York.

Williamson, Oliver E. (1985). The Economic Institutions of Capitalism. The Free Press, New York.

Wolfers, Jutin (2006). Did Unilateral Divorce Laws Raise Divorce Rates? A Reconciliation and New Results. *American Economic Review*, 96(5), 1802-1820.

ONLINE APPENDIX

Long-term Contracts.

We now briefly show that the use of a long-term contract cannot improve the outcome. Suppose that Alice and Bob retain individual asset ownership at date 0, but commit to a contract that specifies the price at which they transact at date 4. If the contract is only structured as an option without commitment, then it has no effect at all. However, if the contract is binding, then the two partners face a similar renegotiation game as under joint asset ownership: If they want to switch partners, they cannot do so without the consent of their original trading partner. This in turn implies that a long-term contract cannot prevent retention externalities.

Moreover, the division of surplus in equilibrium is determined by the bargaining outcome. The only difference between a long-term contract and joint asset ownership is *how* the surplus is split between the two partners. Under joint asset ownership, Alice and Bob each get a constant fraction of the profits, as defined by α and β. With a long-term contract the partners agree on a pre-specified price (or pricing formula) that determines the division of surplus. What matters for the model is not the actual distribution at date 4, but the expected distribution at dates 2 and 3. Suppose Bob (upstream) sells the input to Alice (downstream). Let $\widetilde{v}$ be the value of the input for the buyer (Alice) at date 4, and $\widetilde{c}$ be the cost of the seller (Bob). Their joint surplus is then given by $y = \widetilde{v} - \widetilde{c}$. We conveniently denote the joint distribution of $\widetilde{v}$ and $\widetilde{c}$ by $\Omega_{vc}(\widetilde{v}, \widetilde{c})$, so that $\int y d\Omega_{in}(y) = \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$. Let λ be the transfer price specified in the long-term contract. This price can only be made contingent on verifiable information, i.e., on the realizations of $\widetilde{v}$ and $\widetilde{c}$ at date 4.

With a constant inside prospect π, it is easy to see that the two partners agree on a unique transfer price λ^* that allows them to split the expected surplus according to the bargaining outcome.[30] Alternatively, they can define a flexible pricing schedule that implements the bar-

[30]Specifically, we have $\alpha^* \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c}) = \int (\widetilde{v} - \lambda^*) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$, or equivalently, $\lambda^* = \int \widetilde{v} d\Omega_{vc}(\widetilde{v}, \widetilde{c}) - \alpha^* \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$.

gaining outcome. The flexible pricing schedule must then satisfy $\alpha^* y = \widetilde{v} - \lambda^*$, which requires $\lambda^* = (1 - \alpha^*)\widetilde{v} + \alpha^* \widetilde{c}$. However, a long-term contract only affects the means through which the total surplus is split, and not the division of surplus itself. Thus, long-term contracts generate exactly the same ex-ante utilities as joint asset ownership. This also implies that leaving a partner, and thus breaking the long-term contract, requires the same compensation as under joint asset ownership, thus leading to the same retention externalities.

Alternative Ownership Structures.

We focus on individual and joint asset ownership as the only possible ownership structures. We now briefly explain why we can safely ignore all other ownership structures.

The main alternative ownership structure is full asset ownership in the hands of one of the two partners. With ex-ante symmetric partners, it does not matter which partner owns both assets; w.l.o.g. we assume it is Alice. It is easy to verify that whenever Alice finds an alternative partner and Bob does not, then the model behaves just like under individual asset ownership. And if Bob finds an alternative partner and Alice does not, then the model behaves just like under joint asset ownership. We show that either individual or joint asset ownership is optimal (see Proposition 1); thus, mixing individual and joint ownership is never optimal.

In fact, asymmetric asset ownership creates additional inefficiencies when Alice controls the two assets. If both Alice and Bob find alternative partners, then Alice can hold up Bob before releasing his asset. Thus, Bob would have to share some of the profits from his new partnership with Dora, which is clearly inefficient. At the ex-ante stage, Bob would also not relinquish his asset for free. In fact, Alice would have to give Bob a larger profit share ex-ante, which would lead to further inefficiencies. Asymmetric ownership is therefore never optimal within the present context.

From Proposition 1 it is immediate that randomization among the symmetric ownership structures is also suboptimal. Giving ownership to outsiders is not optimal in our model either, since owner-managers want to retain maximal effort incentives. Moreover, the type of outside ownership suggested by Gans (2005) is not an equilibrium, because we allow asset owners to coordinate on joint asset ownership. Finally, because there are no sequential investments in our model, there is no role for options on ownership as in Nöldeke and Schmidt (1998).

Joint Production.

The optimal effort levels, denoted $e_A^*(\alpha)$ and $e_B^*(\beta)$, are characterized by the first-order conditions

$$\alpha\mu'(e_A e_B)e_B\pi = c'(e_A) \tag{1}$$

$$\beta\mu'(e_A e_B)e_A\pi = c'(e_B). \tag{2}$$

Using $\beta = 1-\alpha$, the joint surplus $V \equiv U_A + U_B$ is given by

$$V(\alpha;\pi) = \mu(e_A^*(\alpha)e_B^*(\alpha))\pi - c(e_A^*(\alpha)) - c(e_B^*(\alpha)).$$

The jointly optimal profit share α^J satisfies the first-order condition

$$\pi\mu'(e_A^* e_B^*)\left[\frac{de_A^*}{d\alpha}e_B^* + \frac{de_B^*}{d\alpha}e_A^*\right] = c'(e_A^*)\frac{de_A^*}{d\alpha} + c'(e_B^*)\frac{de_B^*}{d\alpha}. \tag{3}$$

Symmetry implies $de_A^*/d\alpha = -de_B^*/d\alpha$ and $e_A^* = e_B^*$ at $\alpha = 1/2$. Thus, (3) is satisfied for $\alpha = \beta = 1/2$. The solution $\alpha^J = \beta^J = 1/2$ is also unique due to the convexity of $c(e_i)$, $i = A, B$. Thus,

$$\left.\frac{dU_A}{d\alpha}\right|_{\alpha=1/2} = -\left.\frac{dU_B}{d\alpha}\right|_{\alpha=1/2} > 0.$$

Moreover, $e_A^*(0) = e_B^*(1) = 0$. This implies $1/2 < \alpha^{max} = \beta^{max} < 1$.

Now consider the bargaining at date 0, and suppose that Alice gets the profit share $\alpha > 1/2$ with probability $1/2$, and $1-\alpha < 1/2$ otherwise. Alice's expected utility at date 0 is then given by $U_A(\alpha;\pi)/2 + U_A(1-\alpha;\pi)/2$. However, from the above we note that her expected utility is maximized when $\alpha = 1/2$ (symmetric for Bob). Thus, at date 0 both partners agree on splitting the expected joint surplus in half: $\alpha = \beta = 1/2$. □

Renegotiation under Individual Asset Ownership.

W.l.o.g. suppose that only Alice found an alternative partner, Charles. Consider first the case without wealth ($w = 0$). Alice is indifferent between staying (and renegotiating her profit share α) and leaving, if

$$U_A(\alpha_I^*;\pi) = U_A(\widehat{\alpha}_I;\sigma). \tag{4}$$

Recall that Bob and Charles both have zero outside options. The bargaining protocol à la Hart and Mas-Colell (1996) then implies that (4) is satisfied for $\pi = \sigma$. Thus, $\sigma \leq \pi$ implies $U_A(\alpha_I^*;\pi) \geq U_A(\widehat{\alpha}_I;\sigma)$. For $\sigma > \pi$ we have $U_A(\alpha_I^*;\pi) < U_A(\widehat{\alpha}_I;\sigma)$. We define $\widehat{\pi}_I(\sigma) = \sigma$ as the threshold below which Alice is better off leaving Bob ($\pi < \widehat{\pi}_I(\sigma)$).

Now suppose that Alice and Bob have each initial wealth $w > 0$. Let $V_I(\pi,w) \equiv U_A(\alpha_I^*;\pi,w) + U_B(\beta_I^*;\pi,w)$ denote Alice's and Bob's joint surplus under individual asset ownership when staying together. Recall that their joint surplus is maximized in case of joint production when $\alpha_I^* = \beta_I^* = 1/2$. The joint surplus is then given by $2U(\pi)$. Thus, the minimum value of wealth w required to eliminate displacement externalities under individual asset ownership, denoted $\overline{w}_I$, satisfies $V_I(\pi,w) = 2U(\pi)$. Next we characterize the minimum amount of wealth $\underline{w}_I$, which changes the renegotiation outcome. For $\pi \geq \widehat{\pi}_I(\sigma)$ we know that Alice stays with Bob, but profit shares are unbalanced. Bob can then use even small amounts of wealth to buy back some profit shares from Alice, which improves their joint surplus. Thus, $\underline{w}_I = 0$ for $\pi \geq \widehat{\pi}_I(\sigma)$. For $\pi < \widehat{\pi}_I(\sigma)$, Alice leaves the partnership with Bob. For $w \to 0$ Bob cannot retain Alice, and therefore cannot change the renegotiation outcome. Thus, $\underline{w}_I > 0$, where $\underline{w}_I$ satisfies

$U_A(\alpha_I^*; \pi, \underline{w}_I) = U(\widehat{\alpha}_I, \sigma)$.

Proof of Lemma 1.

It follows directly from our previous derivations (see Section "Renegotiation under Individual Asset Ownership" in the Appendix) that the threshold $\widehat{\pi}_I(\sigma, w)$ is defined by

$$U_A(\alpha_I^*; \pi, w) = U_A(\widehat{\alpha}_I; \sigma). \tag{5}$$

Using (5) we can implicitly differentiate $\widehat{\pi}_I(\sigma, w)$ w.r.t. w:

$$\frac{d\widehat{\pi}_I(\sigma, w)}{dw} = -\frac{\frac{dU_A(\alpha_I^*;\pi,w)}{dw}}{\frac{dU_A(\alpha_I^*;\pi,w)}{d\pi}}.$$

Recall that $dU_A(\alpha_I^*; \pi, w)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$. Moreover, applying the Envelope Theorem we find that $dU_A(\alpha_I^*; \pi, w)/d\pi > 0$. Thus, $d\widehat{\pi}_I(\sigma, w)/dw < 0$ for $\underline{w}_I \leq w < \overline{w}_I$. □

Profit Shares under Joint Ownership with Asymmetric Outside Options.

W.l.o.g. suppose that only Alice found an alternative partner, Charles. We denote a coalition by S, with $S \subset 3$. Let $\kappa = (\kappa_A, \kappa_B, \kappa_C)$ be a vector which measures the rate at which utility can be transferred. Moreover, $\eta_T \in V(T)$ denotes the payoff vector for the subcoalition T.

According to Hart (2004), the Maschler-Owen consistent NTU value can be derived by the following procedure: First, for all $i \in S$, let the payoff vector $\mathbf{z} \in \mathbb{R}^S$ satisfy

$$\kappa_i z_i = \frac{1}{|S|}\left[v_\kappa(S) - \sum_{j \in S\setminus i} \kappa_j \eta_{S\setminus i}(j) + \sum_{j \in S\setminus i} \kappa_i \eta_{S\setminus j}(i)\right],$$

where the maximum possible value $v_\kappa(S)$ is defined by

$$v_\kappa(S) = \sup \left\{ \sum_{i \in S} \kappa_i U_i : (U_i)_{i \in S} \in V(S) \right\}.$$

Second, if $\mathbf{z}$ is feasible, then the payoff vector is given by $\eta_S = \mathbf{z}$.

The coalition functions for our setting are as follows:

$$\begin{aligned}
V_{\{A\}} &= V_{\{B\}} = V_{\{C\}} = 0 \\
V_{\{A,B\}} &= \left\{ (U_A(\alpha;\pi), U_B(\beta;\pi)) \in \mathbb{R}^{\{A,B\}} : \alpha + \beta \leq 1; \alpha, \beta \geq 0 \right\} \\
V_{\{A,C\}} &= \{0, 0\} \\
V_{\{B,C\}} &= \{0, 0\} \\
V_{\{A,B,C\}} &= \left\{ (U_A(\alpha;\sigma), U_B(\beta;\sigma), U_C(\gamma;\sigma)) \in \mathbb{R}^{\{A,B,C\}} : \alpha + \beta + \gamma \leq 1; \alpha, \beta, \gamma \geq 0 \right\},
\end{aligned}$$

where $V_{\{A,C\}} = \{0, 0\}$ follows from the fact that Alice cannot leave without Bob's consent under joint ownership. Note that the bargaining outcome must satisfy $\alpha^* \in (0, \alpha^{\max})$ and $\beta^* \in (0, \beta^{\max})$ for the Alice-Bob coalition, and $\alpha^* \in (0, \alpha^{\max})$, $\beta^* \in (0, \beta^{\max})$, and $\gamma^* \in (0, \gamma^{\max})$ for the grand coalition (Alice, Bob, and Charles). Thus, $dU_A/d\alpha > 0$, $dU_B/d\beta > 0$, and $dU_C/d\gamma > 0$ for the relevant values of α, β, and γ. This implies that the inverse of each utility function exists. We define $\alpha(U_A) \equiv U_A^{-1}(\alpha)$, $\beta(U_B) \equiv U_B^{-1}(\beta)$, and $\gamma(U_C) \equiv U_C^{-1}(\gamma)$. Pareto efficiency then requires

$$\begin{aligned}
\alpha(U_A) + \beta(U_B) &= 1 \quad \text{for } \overline{V}_{\{A,B\}} \\
\alpha(U_A) + \beta(U_B) + \gamma(U_C) &= 1 \quad \text{for } \overline{V}_{\{A,B,C\}}.
\end{aligned}$$

The payoffs for the single-player coalitions are given by $\eta_1(A) = \eta_1(B) = \eta_1(C) = 0$. For the two-player coalitions, the equilibrium payoffs satisfy the Nash bargaining solution. Due to symmetry, the payoffs are given by

$$\begin{aligned}
\eta_2(A, B) &= (U(\pi), U(\pi)) \\
\eta_2(A, C) &= (0, 0) \\
\eta_2(B, C) &= (0, 0)\,.
\end{aligned}$$

It remains to derive the payoff vector $\eta_3(A, B, C)$ for the hyperplane game. For a vector $\mathbf{z} = (z_A, z_B, z_C)$ the equation of the hyperplane is

$$\alpha'(U_A)z_A + \beta'(U_B)z_B + \gamma'(U_C)z_C = r, \tag{6}$$

where

$$r = \alpha'(U_A)U_A + \beta'(U_B)U_B + \gamma'(U_C)U_C. \tag{7}$$

Using the payoffs for the two-player coalitions, we can now define the equilibrium payoffs for the grand coalition:

$$\begin{aligned}
\eta_3(A) &= U_A(\alpha; \sigma) = \frac{1}{3}\left[U(\pi) + z_A\right] \\
\eta_3(B) &= U_B(\beta; \sigma) = \frac{1}{3}\left[U(\pi) + z_B\right] \\
\eta_3(C) &= U_C(\gamma; \sigma) = \frac{1}{3}z_C,
\end{aligned}$$

where, using (7),

$$\begin{aligned} z_A &= \frac{1}{\alpha'(U_A)}\left[r - \beta'(U_B)\cdot 0 - \gamma'(U_C)\cdot 0\right] = \frac{r}{\alpha'(U_A)} \\ z_B &= \frac{1}{\beta'(U_B)}\left[r - \alpha'(U_A)\cdot 0 - \gamma'(U_C)\cdot 0\right] = \frac{r}{\beta'(U_B)} \\ z_C &= \frac{1}{\gamma'(U_C)}\left[r - \alpha'(U_A)U(\pi) - \beta'(U_B)U(\pi)\right]. \end{aligned}$$

Using the Inverse Function Theorem we get $\alpha'(U_A) = (dU_A/d\alpha)^{-1}$, $\beta'(U_B) = (dU_B/d\beta)^{-1}$, and $\gamma'(U_C) = (dU_C/d\gamma)^{-1}$. The equations for the fixed point for the grand coalition are thus given by

$$U_A(\alpha;\sigma) = \frac{1}{3}\left[U(\pi) + r\frac{dU_A(\alpha;\sigma)}{d\alpha}\right] \tag{8}$$

$$U_B(\beta;\sigma) = \frac{1}{3}\left[U(\pi) + r\frac{dU_B(\beta;\sigma)}{d\beta}\right] \tag{9}$$

$$U_C(\gamma;\sigma) = \frac{1}{3}\frac{dU_C(\gamma;\sigma)}{d\gamma}\left[r - U(\pi)\left[\left(\frac{dU_A(\alpha;\sigma)}{d\alpha}\right)^{-1} + \left(\frac{dU_B(\beta;\sigma)}{d\beta}\right)^{-1}\right]\right], \tag{10}$$

where, using (7),

$$r = U_A(\alpha;\sigma)\left(\frac{dU_A(\alpha;\sigma)}{d\alpha}\right)^{-1} + U_B(\beta;\sigma)\left(\frac{dU_B(\beta;\sigma)}{d\beta}\right)^{-1} + U_C(\gamma;\sigma)\left(\frac{dU_C(\gamma;\sigma)}{d\gamma}\right)^{-1}.$$

The equilibrium payoff vector $\eta_3(A,B,C) = (\widehat{U}_A(\alpha;\sigma), \widehat{U}_B(\beta;\sigma), \widehat{U}_C(\gamma;\sigma))$ thus satisfies the system of three equations, (8), (9), and (10), which also defines the equilibrium profit shares $\widehat{\alpha}_J$, $\widehat{\beta}_J$, and $\widehat{\gamma}_J$.

Renegotiation under Joint Asset Ownership.

W.l.o.g. suppose that only Alice found an alternative partner (Charles). We first consider the case without wealth ($w = 0$). Alice will then stay with Bob under joint asset ownership with an equal split of profits if

$$U_A(\pi) \geq U_A(\widehat{\alpha}_J; \sigma). \tag{11}$$

Note that (11) is never satisfied when $\pi = 0$ and $\sigma > 0$. Using the Envelope Theorem one can show that $dU_A(\pi)/d\pi > 0$. Moreover, $\lim_{\pi \to \infty} U_A(\pi) = \infty > U_A(\widehat{\alpha}_J; \sigma)$ for any finite σ. Thus, there exists a threshold $\widehat{\pi}_J(\sigma)$ such that (11) is satisfied for $\pi \geq \widehat{\pi}_J(\sigma)$. Now consider briefly the case where both Alice and Bob found alternative partners (symmetric outside options). They then stay together if $U(\pi) \geq U(\sigma)$, which is equivalent to $\pi \geq \sigma$. Recall that $U(\sigma) > U_A(\widehat{\alpha}_J; \sigma)$ for all $\sigma > 0$ because $\widehat{\beta}_J > 0$ and $e_B^* = 0$ in case of asymmetric outside options. Thus, $\widehat{\pi}_J(\sigma) < \sigma$.

We can now consider the case where Alice and Bob have each some initial wealth $w > 0$. The minimum value of wealth w required to eliminate retention externalities under joint asset ownership, denoted $\overline{w}_J$, ensures that Alice can fully compensate Bob without offering him an equity stake in the new partnership with Charles. Thus, $\overline{w}_J$ satisfies $\widehat{\beta}_J(w) = 0$. It remains to characterize the minimum amount of wealth $\underline{w}_J$, which changes the renegotiation outcome. For $\pi \geq \widehat{\pi}_J(\sigma)$ we know that Alice stays with Bob. For $w \to 0$ Alice cannot buy herself free, so the renegotiation outcome does not change. Thus, $\underline{w}_J = 0$ for $\pi \geq \widehat{\pi}_J(\sigma)$. For $\pi < \widehat{\pi}_J(\sigma)$, Alice leaves Bob, but needs to offer him an equity stake in the new partnership with Charles. Alice can then use even small amounts of wealth to buy back some equity from Bob, which improves Alice's expected utility when partnering with Charles. Consequently, $\underline{w}_J > 0$ for $\pi < \widehat{\pi}_J(\sigma)$.

Proof of Lemma 2.

From our previous derivations (see Section "Renegotiation under Joint Asset Ownership" in the Appendix), we can immediately infer that the threshold $\widehat{\pi}_J(\sigma, w)$ is defined by

$$U(\pi) = U_A(\widehat{\alpha}_J; \sigma, w). \tag{12}$$

Using (12) we can implicitly differentiate $\widehat{\pi}_J(\sigma, w)$ w.r.t. w:

$$\frac{d\widehat{\pi}_J(\sigma, w)}{dw} = \frac{\frac{dU_A(\widehat{\alpha}_J;\sigma,w)}{dw}}{\frac{dU(\pi)}{d\pi}}.$$

We know that $dU_A(\widehat{\alpha}_J; \sigma, w)/dw > 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Furthermore, using the Envelope Theorem it is straightforward to show that $dU(\pi)/d\pi > 0$. Consequently, $d\widehat{\pi}_J(\sigma, w)/dw > 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Finally, recall that Alice and Bob agree on $\alpha^* = \beta^* = 1/2$ at date 0 under joint ownership. And because $\alpha^* = \beta^* = 1/2$ also maximizes total surplus, renegotiation does not change the equity allocation when the partner with the better outside option stays with his original partner. $\square$

Proof of Proposition 1.

We focus on the case with asymmetric outside options because only then the ownership structure matters. Moreover, maximizing a partner's expected utility at date 0 is equivalent to maximizing the joint surplus of Alice and Bob.

We first derive the cutoff $\widehat{\pi}_V(\sigma)$, so that staying together with $\alpha^* = \beta^* = 1/2$ is jointly efficient for $\pi \geq \widehat{\pi}_V(\sigma)$, and dissolving the partnership is jointly efficient for $\pi < \widehat{\pi}_V(\sigma)$. W.l.o.g. suppose that only Alice found an alternative partner at date 2 (the case where only Bob found an alternative partner is symmetric). The joint surplus in case of joint production with $\alpha^* = \beta^* = 1/2$, is given by $2U(\pi)$. When Alice leaves, the joint surplus of Alice and Bob is maximized when Bob, as unproductive partner, does not get a stake in the new Alice-Charles partnership. The joint surplus is then given by $U_A(\widehat{\alpha};\sigma)$, where $\widehat{\alpha}$ is Alice's equity share in the new partnership with Charles. Thus, staying together (with $\alpha^*=\beta^*=1/2$) and dissolving the partnership are both jointly efficient if

$$2U(\pi)=U_A(\widehat{\alpha};\sigma). \tag{13}$$

Recall that $dU(\pi)/d\pi>0$. Moreover, note that $U(0)=0$ and $\lim_{\pi\to\infty}U(\pi)=\infty>U_A(\widehat{\alpha};\sigma)$ for any finite σ. Thus, there exists a threshold $\widehat{\pi}_V(\sigma)$, defined by (13), such that $2U(\pi)\geq U_A(\widehat{\alpha};\sigma)$ for $\pi\geq\widehat{\pi}_V(\sigma)$, and $2U(\pi)<U_A(\widehat{\alpha};\sigma)$ for $\pi<\widehat{\pi}_V(\sigma)$. Using (13) we can implicitly differentiate $\widehat{\pi}_V(\sigma)$ w.r.t. σ:

$$\frac{d\widehat{\pi}_V(\sigma)}{d\sigma}=\frac{\frac{dU_A(\widehat{\alpha};\sigma)}{d\sigma}}{\frac{dU(\pi)}{d\pi}}.$$

Using the Envelope Theorem we can show that $dU_A(\widehat{\alpha};\sigma)/d\sigma>0$ and $dU(\pi)/d\pi>0$. Thus, $d\widehat{\pi}_V(\sigma)/d\sigma>0$.

Suppose that $\pi<\widehat{\pi}_V(\sigma)$, so that dissolving the partnership is jointly optimal. Under individual asset ownership, Alice would leave if $\pi<\widehat{\pi}_I(\sigma,w)$, where according to Lemma 1, $\widehat{\pi}_I(\sigma,w)$ is defined by

$$U_A(\alpha_I^*;\pi,w)=U_A(\widehat{\alpha}_I;\sigma). \tag{14}$$

Note that $2U(\pi)>U_A(\alpha_I^*;\pi,w)$ for $w<\overline{w}_I$, whereas the right-hand sides of (13) and (14) are identical. Thus, $\widehat{\pi}_V(\sigma)<\widehat{\pi}_I(\sigma)$. This implies that individual asset ownership is optimal for $\pi<\widehat{\pi}_V(\sigma)$ as it always ensures the jointly efficient dissolution of the partnership in case of asymmetric outside options.

Now suppose that $\pi\geq\widehat{\pi}_V(\sigma)$, so that staying together with $\alpha^*=\beta^*=1/2$ is jointly optimal in case of asymmetric outside options. Under joint asset ownership, Alice stays (with $\alpha^*=\beta^*=1/2$) if $\pi\geq\widehat{\pi}_J(\sigma)$. Recall from Lemma 2 that $\widehat{\pi}_J(\sigma,w)$ is defined by

$$U(\pi)=U_A(\widehat{\alpha}_J;\sigma,w). \tag{15}$$

To show that $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_V(\sigma)$ for $w < \overline{w}_J$, we define $\widehat{\pi}_J^V(\sigma)$ as the value of π under joint asset ownership where staying together (with $\alpha^* = \beta^* = 1/2$) and dissolving the partnership (with $\widehat{\beta}_J > 0$) lead to the same joint surplus:

$$2U(\pi) = U_A(\widehat{\alpha}_J; \sigma, w) + U_B(\widehat{\beta}_J; \sigma, w). \tag{16}$$

Note that $U_A(\widehat{\alpha}; \sigma) > U_A(\widehat{\alpha}_J; \sigma, w) + U_B(\widehat{\beta}_J; \sigma, w)$ for $w < \overline{w}_J$, whereas the left-hand sides of (13) and (16) are identical. Thus, $\widehat{\pi}_J^V(\sigma, w) < \widehat{\pi}_V(\sigma)$. Moreover, we can write (16) as

$$U(\pi) + \underbrace{U(\pi) - U_B(\widehat{\beta}_J; \sigma, w)}_{\equiv \chi} = U_A(\widehat{\alpha}_J; \sigma, w), \tag{17}$$

where, according to the Maschler-Owen consistent NTU value, $\chi < 0$ (otherwise Bob would not release his asset). Thus, the left-hand side of (17) is smaller than the left-hand side of (15), while their right-hand sides are identical. Hence, $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_J^V(\sigma, w)$. This implies that $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_V(\sigma)$. Thus, joint asset ownership is optimal for $\pi \geq \widehat{\pi}_V(\sigma)$ as it always preserves the partnership with $\alpha^* = \beta^* = 1/2$.

We can now identify the optimal asset ownership for different values of $\pi \in \{\pi_L, \pi_H\}$ and $w < \max\{\overline{w}_I, \overline{w}_J\}$. From the above we can immediately infer that choosing individual asset ownership at date 0 is always optimal when $\pi_L, \pi_H < \widehat{\pi}_V(\sigma)$. Likewise, joint asset ownership is always optimal when $\pi_L, \pi_H \geq \widehat{\pi}_V(\sigma)$.

Next we derive the optimal asset ownership for $\pi_L < \widehat{\pi}_V(\sigma) < \pi_H$. For this we first derive the expected utilities at date 2 when both partners observe the inside prospect $\pi \in \{\pi_L, \pi_H\}$. Consider individual asset ownership. Let α_I^+ denote the profit share of the partner with the only outside option (asymmetric case), and α_I^- the profit share for the partner without outside option, where $\alpha_I^- = 1 - \alpha_I^+$. Moreover, let $\widehat{\alpha}_I$ denote the equilibrium profit share of the partner with outside option when he leaves. The expected utility of a partner a date 2 is then given by

$$EU_I(\pi, \sigma, w) = q^2 \max\{U(\pi), U(\sigma)\} + (1-q)^2 U(\pi) + q(1-q) V_I(\pi, \sigma, w), \qquad (18)$$

where

$$V_I(\pi, \sigma, w) = \begin{cases} U(\widehat{\alpha}_I; \sigma) & \text{if } \pi < \widehat{\pi}_I(\sigma) \\ U(\alpha_I^+; \pi, w) + U(\alpha_I^-; \pi, w) & \text{if } \pi \geq \widehat{\pi}_I(\sigma) \end{cases}$$

is the total expected utility of a partner in case of asymmetric outside options.

Now consider joint asset ownership. Let $\widehat{\alpha}_J$ denote the new profit share of the partner with the only outside option when leaving the partnership, and $\widehat{\beta}_J$ the profit share of his former partner as compensation. The expected utility of a partner at date 2 is then given by

$$EU_J(\pi, \sigma) = q^2 \max\{U(\pi), U(\sigma)\} + (1-q)^2 U(\pi) + q(1-q) V_J(\pi, \sigma, w), \qquad (19)$$

where

$$V_J(\pi, \sigma, w) = \begin{cases} U(\widehat{\alpha}_J; \sigma, w) + U(\widehat{\beta}_J; \sigma, w) & \text{if } \pi < \widehat{\pi}_J(\sigma) \\ 2U(\pi) & \text{if } \pi \geq \widehat{\pi}_J(\sigma) \end{cases}$$

is the total expected utility of a partner in case of asymmetric outside options.

We can now write the expected utility of a partner at date 0 under individual asset ownership ($EU_I(p)$) and joint asset ownership ($EU_J(p)$) as

$$EU_k(p) = pEU_k(\pi_H, \sigma, w) + (1-p) EU_k(\pi_L, \sigma, w), \quad k = I, J$$

where $EU_I(\pi, \sigma, w)$ and $EU_J(\pi, \sigma, w)$ are defined by (18) and (19), respectively. Thus, both partners agree on joint asset ownership at date 0 when $EU_J(p) \geq EU_I(p)$, which is equivalent to

$$p \geq \widehat{p} \equiv \frac{V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w)}{V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w) + V_J(\pi_H, \sigma, w) - V_I(\pi_H, \sigma, w)}.$$

where $V_J(\pi_H, \sigma, w) - V_I(\pi_H, \sigma, w) > 0$ and $V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w) > 0$ for $\pi_L < \widehat{\pi}_V(\sigma) < \pi_H$. □

Alternative Bargaining Protocols.

If both partners have zero outside options, they are perfectly symmetric. Any reasonable bargaining solution then suggests an equal split of surplus. Similarly, if Alice and Bob both found alternative partners, then we have two pairs of symmetric partners. Again we note that an equal split of surplus is the most reasonable bargaining outcome. Alternative bargaining protocols therefore only matter for the case of asymmetric outside options. We distinguish between the bargaining games under individual versus joint asset ownership.

Consider first the bargaining game under joint asset ownership with binding wealth constraints, where Alice wants to leave Bob to partner with Charles. Because the agreement of all three parties is required, any reasonable bargaining involves trilateral bargaining. While there may be many bargaining protocols that affect the distribution of rents between the three parties, the key insight is that the critical threshold $\widehat{\pi}_J(\sigma, w)$ from Lemma 2 does not depend on the specific distribution of these rents. This threshold only depends on the feasibility of obtaining an agreement between Alice, Bob and Charles that satisfies all three participation constraints. Specifically, at $\pi = \widehat{\pi}_J(\sigma, w)$ both Alice and Bob are indifferent between dissolving their partnership and staying together (each getting $U(\pi)$), while Charles receives the minimum equity stake $\gamma = 1 - \alpha^{\max}$. For any $\pi > \widehat{\pi}_J(\sigma, w)$ it is impossible to get a tripartite agreement, and for any $\pi \leq \widehat{\pi}_J(\sigma, w)$ it is always possible get such an agreement. As a consequence, the specific bargaining protocol actually does not matter for the partners' decision to stay together or to do a buyout.

Under individual asset ownership with binding wealth constraints we know from Lemma 1 that there exists a critical threshold $\widehat{\pi}_I(\sigma, w)$, such that Alice leaves Bob whenever $\pi < \widehat{\pi}_I(\sigma, w)$, and stays whenever $\pi \geq \widehat{\pi}_I(\sigma, w)$. Again we argue that reasonable alternative bargaining protocols may generate different utilities, but the critical threshold remains unaffected. One important restriction of the bargaining protocol by Hart and Mas-Colell (1996) is that at any point in time only one party can make an offer. Consider relaxing this assumption, and suppose that there can be simultaneous offers. In particular assume that the unique partner (Alice) can hold an auction for offers from the non-unique partners (Bob and Charles). Such an auction game results in a standard Bertrand pricing. It is easy to show that these Bertrand offers are more favorable to Alice than the bargaining outcome under the Hart and Mas-Colell protocol. However, since the auction is always won by the player with the highest valuation, it continues to be true that Alice teams up with Bob whenever $\pi \geq \widehat{\pi}_I(\sigma, w)$, and with Charles whenever $\pi < \widehat{\pi}_I(\sigma, w)$. Again we find that the critical threshold $\widehat{\pi}_I(\sigma, w)$ remains unaffected by the specific bargaining protocol.

Proof of Proposition 2.

Consider individual asset ownership. At date 1 partner $i = A, B$ chooses his specific investment r_i to maximize his expected utility:[31]

$$\begin{aligned} EU_I(r_i, r_j) \;=\; & p(r_i, r_j)\left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_I(\pi_H, \sigma, w)\right] \\ & + (1 - p(r_i, r_j))\left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_I(\pi_L, \sigma, w)\right] - \psi(r_i), \end{aligned}$$

[31] Note that $U(\sigma) > U(\pi_L)$ when Alice and Bob each found an alternative partner; thus, $\max\{U(\pi_L), U(\sigma)\} = U(\sigma)$.

where $j \in \{A, B\}$ and $j \neq i$. The equilibrium investment levels $r^*_{A(I)}(w)$ and $r^*_{B(I)}(w)$ under individual asset ownership are then characterized by the first-order conditions:

$$\frac{\partial p(r_A, r_B)}{\partial r_i} \Phi_I(w) = \psi'(r_i), \quad i = A, B,$$

where, using $V_I(\pi_L, \sigma, w) = U(\sigma)$,

$$\begin{aligned} \Phi_I(w) &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_I(\pi_H, \sigma, w)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) U(\sigma)\right]. \end{aligned}$$

Because Alice and Bob are symmetric at date 1, their investment levels $r^*_{A(I)}(w)$ and $r^*_{B(I)}(w)$ must be also symmetric in equilibrium. We define $r^*_I(w) \equiv r^*_{A(I)}(w) = r^*_{B(I)}(w)$ as the equilibrium relation-specific investment of a partner under individual asset ownership.

Likewise, the expected utility of partner $i = A, B$ at date 1 under joint asset ownership is given by

$$\begin{aligned} EU_J(p_i, p_j) &= p(r_i, r_j) \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_J(\pi_H, \sigma, w)\right] \\ &\quad + (1 - p(r_i, r_j)) \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_J(\pi_L, \sigma, w)\right] - \psi(r_i). \end{aligned}$$

The following first-order conditions define the equilibrium investment levels $r^*_{A(J)}(w)$ and $r^*_{B(J)}(w)$ under joint asset ownership:

$$\frac{\partial p(r_A, r_B)}{\partial r_i} \Phi_J(w) = \psi'(r_i) \quad i = A, B,$$

where, using $V_J(\pi_H, \sigma, w) = 2U(\pi_H)$,

$$\begin{aligned}\Phi_J(w) &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)2U(\pi_H)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_J(\pi_L, \sigma, w)\right].\end{aligned}$$

Again, the Nash equilibrium is symmetric; we thus define $r^*_J(w) \equiv r^*_{A(J)}(w) = r^*_{B(J)}(w)$ as the equilibrium relation-specific investment of a partner under joint asset ownership.

Next, we define

$$\begin{aligned}F &\equiv \frac{\partial p(r_A, r_B)}{\partial r_A}\Phi_k(w) - \psi'(r_A) = 0 \\ G &\equiv \frac{\partial p(r_A, r_B)}{\partial r_B}\Phi_k(w) - \psi'(r_B) = 0,\end{aligned}$$

where $k \in \{I, J\}$. Applying Cramer's Rule we get

$$\frac{dr^*_{A(k)}(w)}{dw} = \frac{\det(X_1)}{\det(X_2)},$$

where

$$X_1 = \begin{pmatrix} -\frac{\partial F}{\partial w} & \frac{\partial F}{\partial r_B} \\ -\frac{\partial G}{\partial w} & \frac{\partial G}{\partial r_B} \end{pmatrix} \qquad X_2 = \begin{pmatrix} \frac{\partial F}{\partial r_A} & \frac{\partial F}{\partial r_B} \\ \frac{\partial G}{\partial r_A} & \frac{\partial G}{\partial r_B} \end{pmatrix}.$$

Because $U_i(\cdot)$, $i = A, B$, is concave, X_2 must be negative definite, so that $\det(X_2) > 0$. Thus, $dr^*_{A(k)}(w)/dw > 0$ if

$$\det(X_1) = -\frac{\partial F}{\partial w}\frac{\partial G}{\partial r_B} + \frac{\partial G}{\partial w}\frac{\partial F}{\partial r_B} > 0.$$

The second-order condition for $r^*_{B(k)}(w)$ implies $\partial G/\partial r_B < 0$. Moreover,

$$\frac{\partial F}{\partial r_B} = \frac{\partial^2 p(\cdot)}{\partial r_A \partial r_B}\Phi_k(w)$$

and

$$\frac{\partial F}{\partial w} = \frac{\partial p(\cdot)}{\partial r_A}\frac{d\Phi_k(w)}{dw} \qquad \frac{\partial G}{\partial w} = \frac{\partial p(\cdot)}{\partial r_B}\frac{d\Phi_k(w)}{dw},$$

where

$$\begin{aligned} \frac{d\Phi_I(w)}{dw} &= q(1-q)\frac{dV_I(w,\pi_H,\sigma)}{dw} \\ \frac{d\Phi_J(w)}{dw} &= -q(1-q)\frac{dV_J(w,\pi_L,\sigma)}{dw}. \end{aligned}$$

For individual asset ownership, recall that $dV_I(w,\pi_H,\sigma)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$, which implies that $\partial F/\partial w > 0$ and $\partial G/\partial w > 0$ for $\underline{w}_I \leq w < \overline{w}_I$. Thus, $dr^*_{A(I)}(w)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$ and $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$, where κ is the lower bound of the cross-partial satisfying $\det(X_1) = 0$. Symmetry implies $dr^*_{A(I)}(w)/dw = dr^*_{B(I)}(w)/dw$. For joint asset ownership, recall that $dV_J(w,\pi_L,\sigma)/dw > 0$ for $\underline{w}_J \leq w < \overline{w}_J$, so that $\partial F/\partial w < 0$ and $\partial G/\partial w < 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Thus, $dr^*_{A(J)}(w)/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$ and $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$. Due to symmetry, $dr^*_{A(J)}(w)/dw = dr^*_{B(J)}(w)/dw$.

For $w \geq \max\{\overline{w}_I, \overline{w}_J\}$ we know that $V_I(w,\pi_H,\sigma) = 2U(\pi_H)$ (individual ownership), and $V_J(w,\pi_L,\sigma) = U(\sigma)$ (joint ownership). Thus, we have $\Phi_I(w) = \Phi_J(w)$ for $w \geq \max\{\overline{w}_I, \overline{w}_J\}$, so that $r^*_I(w) = r^*_J(w)$. Furthermore, because $dr^*_I/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$, and $dr^*_J/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$, we can infer that $r^*_J(w) > r^*_I(w)$ for $w < \max\{\overline{w}_I, \overline{w}_J\}$. □

Proof of Lemma 3.

Under individual asset ownership the expected utility of Alice at date 0 is given by

$$\begin{aligned} EU^A_I(p^*,w) &= p^*\left[q^2\max\{U(\pi_H),U(\sigma)\} + (1-q)^2U(\pi_H) + q(1-q)V_I(\pi_H,\sigma,w)\right] \\ &\quad +(1-p^*)\left[q^2U(\sigma) + (1-q)^2U(\pi_L) + q(1-q)V_I(\pi_L,\sigma,w)\right] - \psi(r^*_{A(I)}), \end{aligned}$$

with $p^* \equiv p(r^*_{A(I)}, r^*_{B(I)})$ and $V_I(\pi_L, \sigma, w) = U(\sigma)$. The expected utility of Bob is symmetric. Applying the Envelope Theorem we get

$$\frac{dEU_I^A(p^*, w)}{dw} = \frac{\partial EU_I^A(p^*, w)}{\partial r_{B(I)}} \frac{dr^*_{B(I)}}{dw} + p^* q(1-q) \frac{\partial V_I(\pi_H, \sigma, w)}{\partial w}.$$

Note that $\partial EU_I^A(\cdot)/\partial r_{B(I)} > 0$. We need to consider three cases: (i) $w \leq \underline{w}_I$, (ii) $w > \overline{w}_I$; and (iii), $\underline{w}_I < w \leq \overline{w}_I$. For the first two cases we know that $dr^*_{B(I)}/dw = 0$ and $\partial V_I / \partial w = 0$; thus, $dEU_I^A(p^*, w)/dw = 0$. For $\underline{w}_I < w \leq \overline{w}_I$ we know that $dr^*_{B(I)}/dw > 0$ and $\partial V_I / \partial w > 0$; thus, $dEU_I^A(p^*, w)/dw > 0$. This also implies that $EU_I^A(p^*, w)$ is maximized for $w \geq \overline{w}_I$. $\square$

Proof of Lemma 4.

Under joint asset ownership the expected utility of Alice at date 0 is given by

$$\begin{aligned} EU_J^A(p^*, w) &= p^* \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_J(\pi_H, \sigma, w) \right] \\ &\quad + (1-p^*) \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_J(\pi_L, \sigma, w) \right] - \psi(r^*_{A(I)}), \end{aligned}$$

with $p^* = p(r^*_{A(J)}, r^*_{B(J)})$ and $V_J(\pi_H, \sigma, w) = 2U(\pi_H)$. The expected utility of Bob is symmetric. Applying the Envelope Theorem yields

$$\begin{aligned} \frac{dEU_J^A(p^*, w)}{dw} &= \frac{\partial EU_J^A(p^*, w)}{\partial r_{B(J)}} \frac{dr^*_{B(J)}}{dw} + (1-p^*) q(1-q) \frac{\partial V_J(\pi_L, \sigma, w)}{\partial w} \\ &= \underbrace{\Phi_J(w) \frac{\partial p(\cdot)}{\partial r_{B(J)}} \frac{dr^*_{B(J)}}{dw}}_{\equiv \psi_1} + \underbrace{(1-p^*) q(1-q) \frac{\partial V_J(\pi_L, \sigma, w)}{\partial w}}_{\equiv \psi_2}, \end{aligned}$$

where

$$\Phi_J(w) = \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)2U(\pi_H)\right]$$
$$- \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_J(\pi_L, \sigma, w)\right] > 0.$$

By definition, $\partial p(\cdot)/\partial r_{B(J)} > 0$. Moreover, recall from Proposition 2 that $dr^*_{B(J)}/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Thus, $\psi_1 < 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Furthermore, $\partial V_J(\pi_L, \sigma, w)/\partial w > 0$ for $\underline{w}_J \leq w < \overline{w}_J$, so that $\psi_2 > 0$ for $\underline{w}_J \leq w < \overline{w}_J$. We define w^*_J as the wealth level which satisfies $dEU^A_J(p^*, w)/dw = 0$ for $\underline{w}_J \leq w < \overline{w}_J$, and thus maximizes Alice's expected utility at date 0. Note that $\underline{w}_J \leq w^*_J < \overline{w}_J$ because $dr^*_{B(J)}/dw = 0$ and $\partial V_J(\pi_L, \sigma, w)/\partial w = 0$ for $w < \underline{w}_J$ and $w \geq \overline{w}_J$. To summarize, (i) $dEU^A_J(\cdot)/dw = 0$ for $w \leq \underline{w}_J$, $w \geq \overline{w}_J$, and $w = w^*_J$ (as $\psi_1 + \psi_2 = 0$), (ii) $dEU^A_J(\cdot)/dw > 0$ for $\underline{w}_J < w < w^*_J$ (as $\psi_1 + \psi_2 > 0$); and (iii), $dEU^A_J(\cdot)/dw < 0$ for $w^*_J < w < \overline{w}_J$ (as $\psi_1 + \psi_2 < 0$).

Finally note that $\lim_{\pi_H \to \infty} \Phi_J(w) = \infty$ as $dU(\pi_H)/d\pi_H > 0$ with $\lim_{\pi_H \to \infty} U(\pi_H) = \infty$. This implies that $\lim_{\pi_H \to \infty} \psi_1 = -\infty$ for $\underline{w}_J \leq w < \overline{w}_J$, while $\sup(\psi_2) < \infty$. Thus, there exists a threshold $\widehat{\pi}_H$ such that $dEU^A_J(\cdot)/dw < 0$ for all $\pi_H \geq \widehat{\pi}_H$ and $w \in (\underline{w}_J, \overline{w}_J)$, which implies a corner solution with $w^*_J \leq \underline{w}_J$. □

Proof of Proposition 3.

Suppose $w \geq \overline{w} = \max\{\overline{w}_I, \overline{w}_J\}$. Under individual asset ownership, $V_I(\pi_H, \sigma, w) = 2U(\pi_H)$ for $w \geq \overline{w}_I$. Under joint asset ownership, $V_J(\pi_L, \sigma, w) = V_I(\pi_L, \sigma, w) = U(\widehat{\alpha}_I; \sigma)$ for $w \geq \overline{w}_J$. Moreover, recall from Proposition 2 that $r^*_I(w) = r^*_J(w)$ for all $w \geq \overline{w}$. Thus, $EU_I(r^*_I, w) = EU_J(r^*_J, w)$ for $w \geq \overline{w}$.

Next, recall from Lemma 3 that $dEU_I(\cdot)/dw > 0$ for $\underline{w}_I < w \leq \overline{w}_I$, where $EU_I(\cdot)$ is maximized for $w \geq \overline{w}_I$. Moreover, we know from Lemma 4 that $dEU_J(\cdot)/dw > 0$ for $\underline{w}_J <$

$w < w_J^*$, and $dEU_J(\cdot)/dw < 0$ for $w_J^* < w \leq \overline{w}_J$, where $EU_J(\cdot)$ is maximized when $w = w_J^*$. This implies that $EU_J(\cdot) > EU_I(\cdot)$ for $w \in [w_J^*, \overline{w})$.

Finally we examine whether $EU_I(\cdot) > EU_J(\cdot)$ for some $w < w_J^*$. Suppose $\pi_H \to \pi_L$. We can then immediately see that $r_I^*(w) = r_J^*(w) = 0$, and hence, $EU_I(\cdot) > EU_J(\cdot)$. We define w_0 as the critical wealth level so that $EU_J(\cdot) > EU_I(\cdot)$ for $w \in [w_0, \overline{w})$. Note that $w_0 < w_J^*$ because $EU_J(\cdot) > EU_I(\cdot)$ for $w_J^* \leq w < \overline{w}$. Moreover, $w_0 \geq 0$ because, when π_H is sufficiently high, $EU_J(\cdot) > EU_I(\cdot)$ even for $w = 0$. Thus, joint asset ownership is strictly optimal for $w_0 \leq w < \overline{w}$, with $w_0 \in [0, w_J^*]$. According to Lemma 4, the optimal wealth level is then $w_J^* \in [0, \overline{w})$, with $w_J^* \leq \underline{w}_J$ for all $\pi_H \geq \widehat{\pi}_H$. □

Preferences – Ex-ante Asymmetric Outside Options.

W.l.o.g. we focus on Alice's preference for the allocation of control rights; Bob's preference is symmetric. Let $\overline{q}_i \equiv 1 - q_i$, $i = A, B$. Alice's expected utility at date 0 under joint asset ownership is

$$U_A^J = q_A q_B \frac{\sigma}{2} + (\overline{q}_A q_B + q_A \overline{q}_B + \overline{q}_A \overline{q}_B) \frac{\pi}{2}.$$

Likewise, Alice's expected utility at date 0 under individual asset ownership is

$$U_A^I = q_A q_B \frac{\sigma}{2} + q_A \overline{q}_B \frac{\sigma}{2} + \overline{q}_A \overline{q}_B \frac{\pi}{2}.$$

Alice prefers joint asset ownership if $U_A^J > U_A^I$, which is equivalent to

$$q_A < \widehat{q}_A = \left[\left(\frac{1 - q_B}{q_B} \right) \left(\frac{\sigma - \pi}{\pi} \right) + 1 \right]^{-1}.$$

Moreover, after some simplifications we find that

$$\frac{d\widehat{q}_A}{dq_B} = \left[(1-q_B)\left(\frac{\sigma-\pi}{\pi}\right)+q_B\right]^{-2}\left(\frac{\sigma-\pi}{\pi}\right) > 0$$

$$\frac{d^2\widehat{q}_A}{dq_B^2} = -2\left[(1-q_B)\left(\frac{\sigma-\pi}{\pi}\right)+q_B\right]^{-3}\left(\frac{\sigma-\pi}{\pi^2}\right)(2\pi-\sigma).$$

Note that $d^2\widehat{q}_A/dq_B^2 < 0$ if $\pi > \sigma/2$, and $d^2\widehat{q}_A/dq_B^2 > 0$ if $\pi < \sigma/2$.

Partner Uncertainty and the Dynamic Boundary of the Firm

Thomas Hellmann and Veikko Thiele

— ONLINE APPENDIX —

Long-term Contracts.

We now briefly show that the use of a long-term contract cannot improve the outcome. Suppose that Alice and Bob retain individual asset ownership at date 0, but commit to a contract that specifies the price at which they transact at date 4. If the contract is only structured as an option without commitment, then it has no effect at all. However, if the contract is binding, then the two partners face a similar renegotiation game as under joint asset ownership: If they want to switch partners, they cannot do so without the consent of their original trading partner. This in turn implies that a long-term contract cannot prevent retention externalities.

Moreover, the division of surplus in equilibrium is determined by the bargaining outcome. The only difference between a long-term contract and joint asset ownership is *how* the surplus is split between the two partners. Under joint asset ownership, Alice and Bob each get a constant fraction of the profits, as defined by α and β. With a long-term contract the partners agree on a pre-specified price (or pricing formula) that determines the division of surplus. What matters for the model is not the actual distribution at date 4, but the expected distribution at dates 2 and 3. Suppose Bob (upstream) sells the input to Alice (downstream). Let $\widetilde{v}$ be the value of the input for the buyer (Alice) at date 4, and $\widetilde{c}$ be the cost of the seller (Bob). Their joint surplus is then given by $y = \widetilde{v} - \widetilde{c}$. We conveniently denote the joint distribution of $\widetilde{v}$ and $\widetilde{c}$ by $\Omega_{vc}(\widetilde{v}, \widetilde{c})$, so that $\int y d\Omega_{in}(y) = \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$. Let λ be the transfer price specified in the long-term contract. This price can only be made contingent on verifiable information, i.e., on the realizations of $\widetilde{v}$ and $\widetilde{c}$ at date 4.

With a constant inside prospect π, it is easy to see that the two partners agree on a unique transfer price λ^* that allows them to split the expected surplus according to the bargaining outcome.[1] Alternatively, they can define a flexible pricing schedule that implements the bargaining outcome. The flexible pricing schedule must then satisfy $\alpha^* y = \widetilde{v} - \lambda^*$, which requires $\lambda^* = (1 - \alpha^*)\widetilde{v} + \alpha^* \widetilde{c}$. However, a long-term contract only affects the means through which the total surplus is split, and not the division of surplus itself. Thus, long-term contracts gen-

[1] Specifically, we have $\alpha^* \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c}) = \int (\widetilde{v} - \lambda^*) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$, or equivalently, $\lambda^* = \int \widetilde{v} d\Omega_{vc}(\widetilde{v}, \widetilde{c}) - \alpha^* \int (\widetilde{v} - \widetilde{c}) d\Omega_{vc}(\widetilde{v}, \widetilde{c})$.

erate exactly the same ex-ante utilities as joint asset ownership. This also implies that leaving a partner, and thus breaking the long-term contract, requires the same compensation as under joint asset ownership, thus leading to the same retention externalities.

Alternative Ownership Structures.

We focus on individual and joint asset ownership as the only possible ownership structures. We now briefly explain why we can safely ignore all other ownership structures.

The main alternative ownership structure is full asset ownership in the hands of one of the two partners. With ex-ante symmetric partners, it does not matter which partner owns both assets; w.l.o.g. we assume it is Alice. It is easy to verify that whenever Alice finds an alternative partner and Bob does not, then the model behaves just like under individual asset ownership. And if Bob finds an alternative partner and Alice does not, then the model behaves just like under joint asset ownership. We show that either individual or joint asset ownership is optimal (see Proposition 1); thus, mixing individual and joint ownership is never optimal.

In fact, asymmetric asset ownership creates additional inefficiencies when Alice controls the two assets. If both Alice and Bob find alternative partners, then Alice can hold up Bob before releasing his asset. Thus, Bob would have to share some of the profits from his new partnership with Dora, which is clearly inefficient. At the ex-ante stage, Bob would also not relinquish his asset for free. In fact, Alice would have to give Bob a larger profit share ex-ante, which would lead to further inefficiencies. Asymmetric ownership is therefore never optimal within the present context.

From Proposition 1 it is immediate that randomization among the symmetric ownership structures is also suboptimal. Giving ownership to outsiders is not optimal in our model either, since owner-managers want to retain maximal effort incentives. Moreover, the type of outside ownership suggested by Gans (2005) is not an equilibrium, because we allow asset owners to coordinate on joint asset ownership. Finally, because there are no sequential investments in our model, there is no role for options on ownership as in Nöldeke and Schmidt (1998).

Joint Production.

The optimal effort levels, denoted $e_A^*(\alpha)$ and $e_B^*(\beta)$, are characterized by the first-order conditions

$$\alpha\mu'(e_A e_B)e_B\pi = c'(e_A) \tag{A.1}$$

$$\beta\mu'(e_A e_B)e_A\pi = c'(e_B). \tag{A.2}$$

Using $\beta = 1 - \alpha$, the joint surplus $V \equiv U_A + U_B$ is given by

$$V(\alpha; \pi) = \mu(e_A^*(\alpha)e_B^*(\alpha))\pi - c(e_A^*(\alpha)) - c(e_B^*(\alpha)).$$

The jointly optimal profit share α^J satisfies the first-order condition

$$\pi\mu'(e_A^* e_B^*)\left[\frac{de_A^*}{d\alpha}e_B^* + \frac{de_B^*}{d\alpha}e_A^*\right] = c'(e_A^*)\frac{de_A^*}{d\alpha} + c'(e_B^*)\frac{de_B^*}{d\alpha}. \tag{A.3}$$

Symmetry implies $de_A^*/d\alpha = -de_B^*/d\alpha$ and $e_A^* = e_B^*$ at $\alpha = 1/2$. Thus, (A.3) is satisfied for $\alpha = \beta = 1/2$. The solution $\alpha^J = \beta^J = 1/2$ is also unique due to the convexity of $c(e_i)$, $i = A, B$. Thus,

$$\left.\frac{dU_A}{d\alpha}\right|_{\alpha=1/2} = -\left.\frac{dU_B}{d\alpha}\right|_{\alpha=1/2} > 0.$$

Moreover, $e_A^*(0) = e_B^*(1) = 0$. This implies $1/2 < \alpha^{max} = \beta^{max} < 1$.

Now consider the bargaining at date 0, and suppose that Alice gets the profit share $\alpha > 1/2$ with probability $1/2$, and $1 - \alpha < 1/2$ otherwise. Alice's expected utility at date 0 is then given by $U_A(\alpha; \pi)/2 + U_A(1-\alpha; \pi)/2$. However, from the above we note that her expected utility is maximized when $\alpha = 1/2$ (symmetric for Bob). Thus, at date 0 both partners agree on splitting the expected joint surplus in half: $\alpha = \beta = 1/2$. □

Renegotiation under Individual Asset Ownership.

W.l.o.g. suppose that only Alice found an alternative partner, Charles. Consider first the case without wealth ($w = 0$). Alice is indifferent between staying (and renegotiating her profit share α) and leaving, if

$$U_A(\alpha_I^*; \pi) = U_A(\widehat{\alpha}_I; \sigma). \tag{A.4}$$

Recall that Bob and Charles both have zero outside options. The bargaining protocol à la Hart and Mas-Colell (1996) then implies that (A.4) is satisfied for $\pi = \sigma$. Thus, $\sigma \leq \pi$ implies $U_A(\alpha_I^*; \pi) \geq U_A(\widehat{\alpha}_I; \sigma)$. For $\sigma > \pi$ we have $U_A(\alpha_I^*; \pi) < U_A(\widehat{\alpha}_I; \sigma)$. We define $\widehat{\pi}_I(\sigma) = \sigma$ as the threshold below which Alice is better off leaving Bob ($\pi < \widehat{\pi}_I(\sigma)$).

Suppose that Alice and Bob have each initial wealth $w > 0$. Let $V_I(\pi, w) \equiv U_A(\alpha_I^*; \pi, w) + U_B(\beta_I^*; \pi, w)$ denote Alice's and Bob's joint surplus under individual asset ownership when staying together. Recall that their joint surplus is maximized in case of joint production when $\alpha_I^* = \beta_I^* = 1/2$. The joint surplus is then given by $2U(\pi)$. Thus, the minimum value of wealth w required to eliminate displacement externalities under individual asset ownership, denoted $\overline{w}_I$, satisfies $V_I(\pi, w) = 2U(\pi)$. Next we characterize the minimum amount of wealth $\underline{w}_I$, which changes the renegotiation outcome. For $\pi \geq \widehat{\pi}_I(\sigma)$ we know that Alice stays with Bob, but profit shares are unbalanced. Bob can then use even small amounts of wealth to buy back some profit shares from Alice, which improves their joint surplus. Thus, $\underline{w}_I = 0$ for $\pi \geq \widehat{\pi}_I(\sigma)$. For $\pi < \widehat{\pi}_I(\sigma)$, Alice leaves the partnership with Bob. For $w \to 0$ Bob cannot retain Alice, and therefore cannot change the renegotiation outcome. Thus, $\underline{w}_I > 0$, where $\underline{w}_I$ satisfies $U_A(\alpha_I^*; \pi, \underline{w}_I) = U(\widehat{\alpha}_I, \sigma)$.

Proof of Lemma 1.

It follows directly from our previous derivations (see Section "Renegotiation under Individual Asset Ownership" in the Appendix) that the threshold $\widehat{\pi}_I(\sigma, w)$ is defined by

$$U_A(\alpha_I^*; \pi, w) = U_A(\widehat{\alpha}_I; \sigma). \tag{A.5}$$

Using (A.5) we can implicitly differentiate $\widehat{\pi}_I(\sigma, w)$ w.r.t. w:

$$\frac{d\widehat{\pi}_I(\sigma, w)}{dw} = -\frac{\frac{dU_A(\alpha_I^*;\pi,w)}{dw}}{\frac{dU_A(\alpha_I^*;\pi,w)}{d\pi}}.$$

Recall that $dU_A(\alpha_I^*; \pi, w)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$. Moreover, applying the Envelope Theorem we find that $dU_A(\alpha_I^*; \pi, w)/d\pi > 0$. Thus, $d\widehat{\pi}_I(\sigma, w)/dw < 0$ for $\underline{w}_I \leq w < \overline{w}_I$. □

Profit Shares under Joint Ownership with Asymmetric Outside Options.

W.l.o.g. suppose that only Alice found an alternative partner, Charles. We denote a coalition by S, with $S \subset 3$. Let $\kappa = (\kappa_A, \kappa_B, \kappa_C)$ be a vector which measures the rate at which utility can be transferred. Moreover, $\eta_T \in V(T)$ denotes the payoff vector for the subcoalition T.

According to Hart (2004), the Maschler-Owen consistent NTU value can be derived by the following procedure: First, for all $i \in S$, let the payoff vector $\mathbf{z} \in \mathbb{R}^S$ satisfy

$$\kappa_i z_i = \frac{1}{|S|}\left[v_\kappa(S) - \sum_{j \in S \setminus i} \kappa_j \eta_{S \setminus i}(j) + \sum_{j \in S \setminus i} \kappa_i \eta_{S \setminus j}(i)\right],$$

where the maximum possible value $v_\kappa(S)$ is defined by

$$v_\kappa(S) = \sup\left\{\sum_{i \in S} \kappa_i U_i : (U_i)_{i \in S} \in V(S)\right\}.$$

Second, if $\mathbf{z}$ is feasible, then the payoff vector is given by $\eta_S = \mathbf{z}$.

The coalition functions for our setting are as follows:

$$\begin{aligned}
V_{\{A\}} &= V_{\{B\}} = V_{\{C\}} = 0 \\
V_{\{A,B\}} &= \left\{(U_A(\alpha;\pi), U_B(\beta;\pi)) \in \mathbb{R}^{\{A,B\}} : \alpha+\beta \leq 1; \alpha,\beta \geq 0\right\} \\
V_{\{A,C\}} &= \{0,0\} \\
V_{\{B,C\}} &= \{0,0\} \\
V_{\{A,B,C\}} &= \left\{(U_A(\alpha;\sigma), U_B(\beta;\sigma), U_C(\gamma;\sigma)) \in \mathbb{R}^{\{A,B,C\}} : \alpha+\beta+\gamma \leq 1; \alpha,\beta,\gamma \geq 0\right\},
\end{aligned}$$

where $V_{\{A,C\}} = \{0,0\}$ follows from the fact that Alice cannot leave without Bob's consent under joint ownership. Note that the bargaining outcome must satisfy $\alpha^* \in (0, \alpha^{\max})$ and $\beta^* \in (0, \beta^{\max})$ for the Alice-Bob coalition, and $\alpha^* \in (0, \alpha^{\max})$, $\beta^* \in (0, \beta^{\max})$, and $\gamma^* \in (0, \gamma^{\max})$ for the grand coalition (Alice, Bob, and Charles). Thus, $dU_A/d\alpha > 0$, $dU_B/d\beta > 0$, and $dU_C/d\gamma > 0$ for the relevant values of α, β, and γ. This implies that the inverse of each utility function exists. We define $\alpha(U_A) \equiv U_A^{-1}(\alpha)$, $\beta(U_B) \equiv U_B^{-1}(\beta)$, and $\gamma(U_C) \equiv U_C^{-1}(\gamma)$. Pareto efficiency then requires

$$\begin{aligned}
\alpha(U_A) + \beta(U_B) &= 1 \quad \text{for } \overline{V}_{\{A,B\}} \\
\alpha(U_A) + \beta(U_B) + \gamma(U_C) &= 1 \quad \text{for } \overline{V}_{\{A,B,C\}}.
\end{aligned}$$

The payoffs for the single-player coalitions are given by $\eta_1(A) = \eta_1(B) = \eta_1(C) = 0$. For the two-player coalitions, the equilibrium payoffs satisfy the Nash bargaining solution. Due to symmetry, the payoffs are given by

$$\begin{aligned}
\eta_2(A,B) &= (U(\pi), U(\pi)) \\
\eta_2(A,C) &= (0,0) \\
\eta_2(B,C) &= (0,0)\,.
\end{aligned}$$

It remains to derive the payoff vector $\eta_3(A, B, C)$ for the hyperplane game. For a vector $\mathbf{z} = (z_A, z_B, z_C)$ the equation of the hyperplane is

$$\alpha'(U_A)z_A + \beta'(U_B)z_B + \gamma'(U_C)z_C = r, \tag{A.6}$$

where

$$r = \alpha'(U_A)U_A + \beta'(U_B)U_B + \gamma'(U_C)U_C. \tag{A.7}$$

Using the payoffs for the two-player coalitions, we can now define the equilibrium payoffs for the grand coalition:

$$\begin{aligned}
\eta_3(A) &= U_A(\alpha;\sigma) = \frac{1}{3}\left[U(\pi) + z_A\right] \\
\eta_3(B) &= U_B(\beta;\sigma) = \frac{1}{3}\left[U(\pi) + z_B\right] \\
\eta_3(C) &= U_C(\gamma;\sigma) = \frac{1}{3}z_C,
\end{aligned}$$

where, using (A.7),

$$\begin{aligned}
z_A &= \frac{1}{\alpha'(U_A)}\left[r - \beta'(U_B)\cdot 0 - \gamma'(U_C)\cdot 0\right] = \frac{r}{\alpha'(U_A)} \\
z_B &= \frac{1}{\beta'(U_B)}\left[r - \alpha'(U_A)\cdot 0 - \gamma'(U_C)\cdot 0\right] = \frac{r}{\beta'(U_B)} \\
z_C &= \frac{1}{\gamma'(U_C)}\left[r - \alpha'(U_A)U(\pi) - \beta'(U_B)U(\pi)\right].
\end{aligned}$$

Using the Inverse Function Theorem we get $\alpha'(U_A) = (dU_A/d\alpha)^{-1}$, $\beta'(U_B) = (dU_B/d\beta)^{-1}$, and $\gamma'(U_C) = (dU_C/d\gamma)^{-1}$. The equations for the fixed point for the grand coalition are thus given by

$$U_A(\alpha;\sigma) = \frac{1}{3}\left[U(\pi) + r\frac{dU_A(\alpha;\sigma)}{d\alpha}\right] \tag{A.8}$$

$$U_B(\beta;\sigma) = \frac{1}{3}\left[U(\pi) + r\frac{dU_B(\beta;\sigma)}{d\beta}\right] \tag{A.9}$$

$$U_C(\gamma;\sigma) = \frac{1}{3}\frac{dU_C(\gamma;\sigma)}{d\gamma}\left[r - U(\pi)\left[\left(\frac{dU_A(\alpha;\sigma)}{d\alpha}\right)^{-1} + \left(\frac{dU_B(\beta;\sigma)}{d\beta}\right)^{-1}\right]\right], \tag{A.10}$$

where, using (A.7),

$$r = U_A(\alpha;\sigma)\left(\frac{dU_A(\alpha;\sigma)}{d\alpha}\right)^{-1} + U_B(\beta;\sigma)\left(\frac{dU_B(\beta;\sigma)}{d\beta}\right)^{-1} + U_C(\gamma;\sigma)\left(\frac{dU_C(\gamma;\sigma)}{d\gamma}\right)^{-1}.$$

The equilibrium payoff vector $\eta_3(A,B,C) = (\widehat{U}_A(\alpha;\sigma), \widehat{U}_B(\beta;\sigma), \widehat{U}_C(\gamma;\sigma))$ thus satisfies the system of three equations, (A.8), (A.9), and (A.10), which also defines the equilibrium profit shares $\widehat{\alpha}_J$, $\widehat{\beta}_J$, and $\widehat{\gamma}_J$.

Renegotiation under Joint Asset Ownership.

W.l.o.g. suppose that only Alice found an alternative partner (Charles). We first consider the case without wealth ($w = 0$). Alice will then stay with Bob under joint asset ownership with an equal split of profits if

$$U_A(\pi) \geq U_A(\widehat{\alpha}_J;\sigma). \tag{A.11}$$

Note that (A.11) is never satisfied when $\pi = 0$ and $\sigma > 0$. Using the Envelope Theorem one can show that $dU_A(\pi)/d\pi > 0$. Moreover, $\lim_{\pi\to\infty} U_A(\pi) = \infty > U_A(\widehat{\alpha}_J;\sigma)$ for any finite σ. Thus, there exists a threshold $\widehat{\pi}_J(\sigma)$ such that (A.11) is satisfied for $\pi \geq \widehat{\pi}_J(\sigma)$. Now consider briefly the case where both Alice and Bob found alternative partners (symmetric outside options). They then stay together if $U(\pi) \geq U(\sigma)$, which is equivalent to $\pi \geq \sigma$. Recall that $U(\sigma) > U_A(\widehat{\alpha}_J;\sigma)$ for all $\sigma > 0$ because $\widehat{\beta}_J > 0$ and $e^*_B = 0$ in case of asymmetric outside options. Thus, $\widehat{\pi}_J(\sigma) < \sigma$.

We can now consider the case where Alice and Bob have each some initial wealth $w > 0$. The minimum value of wealth w required to eliminate retention externalities under joint asset ownership, denoted $\overline{w}_J$, ensures that Alice can fully compensate Bob without offering him an equity stake in the new partnership with Charles. Thus, $\overline{w}_J$ satisfies $\widehat{\beta}_J(w) = 0$. It remains to characterize the minimum amount of wealth $\underline{w}_J$, which changes the renegotiation outcome. For $\pi \geq \widehat{\pi}_J(\sigma)$ we know that Alice stays with Bob. For $w \to 0$ Alice cannot buy herself free, so the renegotiation outcome does not change. Thus, $\underline{w}_J = 0$ for $\pi \geq \widehat{\pi}_J(\sigma)$. For $\pi < \widehat{\pi}_J(\sigma)$, Alice leaves Bob, but needs to offer him an equity stake in the new partnership with Charles. Alice can then use even small amounts of wealth to buy back some equity from Bob, which improves Alice's expected utility when partnering with Charles. Consequently, $\underline{w}_J > 0$ for $\pi < \widehat{\pi}_J(\sigma)$.

Proof of Lemma 2.

From our previous derivations (see Section "Renegotiation under Joint Asset Ownership" in the Appendix), we can immediately infer that the threshold $\widehat{\pi}_J(\sigma, w)$ is defined by

$$U(\pi) = U_A(\widehat{\alpha}_J; \sigma, w). \tag{A.12}$$

Using (A.12) we can implicitly differentiate $\widehat{\pi}_J(\sigma, w)$ w.r.t. w:

$$\frac{d\widehat{\pi}_J(\sigma, w)}{dw} = \frac{\frac{dU_A(\widehat{\alpha}_J;\sigma,w)}{dw}}{\frac{dU(\pi)}{d\pi}}.$$

We know that $dU_A(\widehat{\alpha}_J; \sigma, w)/dw > 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Furthermore, using the Envelope Theorem it is straightforward to show that $dU(\pi)/d\pi > 0$. Consequently, $d\widehat{\pi}_J(\sigma, w)/dw > 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Finally, recall that Alice and Bob agree on $\alpha^* = \beta^* = 1/2$ at date 0 under joint ownership. And because $\alpha^* = \beta^* = 1/2$ also maximizes total surplus, renegotiation does not change the equity allocation when the partner with the better outside option stays with his original partner. □

Proof of Proposition 1.

We focus on the case with asymmetric outside options because only then the ownership structure matters. Moreover, maximizing a partner's expected utility at date 0 is equivalent to maximizing the joint surplus of Alice and Bob.

We first derive the cutoff $\widehat{\pi}_V(\sigma)$, so that staying together with $\alpha^* = \beta^* = 1/2$ is jointly efficient for $\pi \geq \widehat{\pi}_V(\sigma)$, and dissolving the partnership is jointly efficient for $\pi < \widehat{\pi}_V(\sigma)$. W.l.o.g. suppose that only Alice found an alternative partner at date 2 (the case where only Bob found an alternative partner is symmetric). The joint surplus in case of joint production with $\alpha^* = \beta^* = 1/2$, is given by $2U(\pi)$. When Alice leaves, the joint surplus of Alice and Bob is maximized when Bob, as unproductive partner, does not get a stake in the new Alice-Charles partnership. The joint surplus is then given by $U_A(\widehat{\alpha}; \sigma)$, where $\widehat{\alpha}$ is Alice's equity share in the new partnership with Charles. Thus, staying together (with $\alpha^* = \beta^* = 1/2$) and dissolving the partnership are both jointly efficient if

$$2U(\pi) = U_A(\widehat{\alpha}; \sigma). \tag{A.13}$$

Recall that $dU(\pi)/d\pi > 0$. Moreover, note that $U(0) = 0$ and $\lim_{\pi\to\infty} U(\pi) = \infty > U_A(\widehat{\alpha}; \sigma)$ for any finite σ. Thus, there exists a threshold $\widehat{\pi}_V(\sigma)$, defined by (A.13), such that $2U(\pi) \geq$

$U_A(\widehat{\alpha};\sigma)$ for $\pi \geq \widehat{\pi}_V(\sigma)$, and $2U(\pi) < U_A(\widehat{\alpha};\sigma)$ for $\pi < \widehat{\pi}_V(\sigma)$. Using (A.13) we can implicitly differentiate $\widehat{\pi}_V(\sigma)$ w.r.t. σ:

$$\frac{d\widehat{\pi}_V(\sigma)}{d\sigma} = \frac{\frac{dU_A(\widehat{\alpha};\sigma)}{d\sigma}}{\frac{dU(\pi)}{d\pi}}.$$

Using the Envelope Theorem we can show that $dU_A(\widehat{\alpha};\sigma)/d\sigma > 0$ and $dU(\pi)/d\pi > 0$. Thus, $d\widehat{\pi}_V(\sigma)/d\sigma > 0$.

Suppose that $\pi < \widehat{\pi}_V(\sigma)$, so that dissolving the partnership is jointly optimal. Under individual asset ownership, Alice would leave if $\pi < \widehat{\pi}_I(\sigma, w)$, where according to Lemma 1, $\widehat{\pi}_I(\sigma, w)$ is defined by

$$U_A(\alpha_I^*;\pi, w) = U_A(\widehat{\alpha}_I;\sigma). \tag{A.14}$$

Note that $2U(\pi) > U_A(\alpha_I^*;\pi, w)$ for $w < \overline{w}_I$, whereas the right-hand sides of (A.13) and (A.14) are identical. Thus, $\widehat{\pi}_V(\sigma) < \widehat{\pi}_I(\sigma)$. This implies that individual asset ownership is optimal for $\pi < \widehat{\pi}_V(\sigma)$ as it always ensures the jointly efficient dissolution of the partnership in case of asymmetric outside options.

Now suppose that $\pi \geq \widehat{\pi}_V(\sigma)$, so that staying together with $\alpha^* = \beta^* = 1/2$ is jointly optimal in case of asymmetric outside options. Under joint asset ownership, Alice stays (with $\alpha^* = \beta^* = 1/2$) if $\pi \geq \widehat{\pi}_J(\sigma)$. Recall from Lemma 2 that $\widehat{\pi}_J(\sigma, w)$ is defined by

$$U(\pi) = U_A(\widehat{\alpha}_J;\sigma, w). \tag{A.15}$$

To show that $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_V(\sigma)$ for $w < \overline{w}_J$, we define $\widehat{\pi}_J^V(\sigma)$ as the value of π under joint asset ownership where staying together (with $\alpha^* = \beta^* = 1/2$) and dissolving the partnership (with $\widehat{\beta}_J > 0$) lead to the same joint surplus:

$$2U(\pi) = U_A(\widehat{\alpha}_J;\sigma, w) + U_B(\widehat{\beta}_J;\sigma, w). \tag{A.16}$$

Note that $U_A(\widehat{\alpha};\sigma) > U_A(\widehat{\alpha}_J;\sigma, w) + U_B(\widehat{\beta}_J;\sigma, w)$ for $w < \overline{w}_J$, whereas the left-hand sides of (A.13) and (A.16) are identical. Thus, $\widehat{\pi}_J^V(\sigma, w) < \widehat{\pi}_V(\sigma)$. Moreover, we can write (A.16) as

$$U(\pi) + \underbrace{U(\pi) - U_B(\widehat{\beta}_J;\sigma, w)}_{\equiv\chi} = U_A(\widehat{\alpha}_J;\sigma, w), \tag{A.17}$$

where, according to the Maschler-Owen consistent NTU value, $\chi < 0$ (otherwise Bob would not release his asset). Thus, the left-hand side of (A.17) is smaller than the left-hand side of

(A.15), while their right-hand sides are identical. Hence, $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_J^V(\sigma, w)$. This implies that $\widehat{\pi}_J(\sigma, w) < \widehat{\pi}_V(\sigma)$. Thus, joint asset ownership is optimal for $\pi \geq \widehat{\pi}_V(\sigma)$ as it always preserves the partnership with $\alpha^* = \beta^* = 1/2$.

We can now identify the optimal asset ownership for different values of $\pi \in \{\pi_L, \pi_H\}$ and $w < \max\{\overline{w}_I, \overline{w}_J\}$. From the above we can immediately infer that choosing individual asset ownership at date 0 is always optimal when $\pi_L, \pi_H < \widehat{\pi}_V(\sigma)$. Likewise, joint asset ownership is always optimal when $\pi_L, \pi_H \geq \widehat{\pi}_V(\sigma)$.

Next we derive the optimal asset ownership for $\pi_L < \widehat{\pi}_V(\sigma) < \pi_H$. For this we first derive the expected utilities at date 2 when both partners observe the inside prospect $\pi \in \{\pi_L, \pi_H\}$. Consider individual asset ownership. Let α_I^+ denote the profit share of the partner with the only outside option (asymmetric case), and α_I^- the profit share for the partner without outside option, where $\alpha_I^- = 1 - \alpha_I^+$. Moreover, let $\widehat{\alpha}_I$ denote the equilibrium profit share of the partner with outside option when he leaves. The expected utility of a partner a date 2 is then given by

$$EU_I(\pi, \sigma, w) = q^2 \max\{U(\pi), U(\sigma)\} + (1-q)^2 U(\pi) + q(1-q) V_I(\pi, \sigma, w), \qquad \text{(A.18)}$$

where

$$V_I(\pi, \sigma, w) = \begin{cases} U(\widehat{\alpha}_I; \sigma) & \text{if } \pi < \widehat{\pi}_I(\sigma) \\ U(\alpha_I^+; \pi, w) + U(\alpha_I^-; \pi, w) & \text{if } \pi \geq \widehat{\pi}_I(\sigma) \end{cases}$$

is the total expected utility of a partner in case of asymmetric outside options.

Now consider joint asset ownership. Let $\widehat{\alpha}_J$ denote the new profit share of the partner with the only outside option when leaving the partnership, and $\widehat{\beta}_J$ the profit share of his former partner as compensation. The expected utility of a partner at date 2 is then given by

$$EU_J(\pi, \sigma) = q^2 \max\{U(\pi), U(\sigma)\} + (1-q)^2 U(\pi) + q(1-q) V_J(\pi, \sigma, w), \qquad \text{(A.19)}$$

where

$$V_J(\pi, \sigma, w) = \begin{cases} U(\widehat{\alpha}_J; \sigma, w) + U(\widehat{\beta}_J; \sigma, w) & \text{if } \pi < \widehat{\pi}_J(\sigma) \\ 2U(\pi) & \text{if } \pi \geq \widehat{\pi}_J(\sigma) \end{cases}$$

is the total expected utility of a partner in case of asymmetric outside options.

We can now write the expected utility of a partner at date 0 under individual asset ownership ($EU_I(p)$) and joint asset ownership ($EU_J(p)$) as

$$EU_k(p) = pEU_k(\pi_H, \sigma, w) + (1-p)EU_k(\pi_L, \sigma, w), \quad k = I, J$$

where $EU_I(\pi, \sigma, w)$ and $EU_J(\pi, \sigma, w)$ are defined by (A.18) and (A.19), respectively. Thus, both partners agree on joint asset ownership at date 0 when $EU_J(p) \geq EU_I(p)$, which is equivalent to

$$p \geq \widehat{p} \equiv \frac{V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w)}{V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w) + V_J(\pi_H, \sigma, w) - V_I(\pi_H, \sigma, w)}.$$

where $V_J(\pi_H, \sigma, w) - V_I(\pi_H, \sigma, w) > 0$ and $V_I(\pi_L, \sigma, w) - V_J(\pi_L, \sigma, w) > 0$ for $\pi_L < \widehat{\pi}_V(\sigma) < \pi_H$. □

Alternative Bargaining Protocols.

If both partners have zero outside options, they are perfectly symmetric. Any reasonable bargaining solution then suggests an equal split of surplus. Similarly, if Alice and Bob both found alternative partners, then we have two pairs of symmetric partners. Again we note that an equal split of surplus is the most reasonable bargaining outcome. Alternative bargaining protocols therefore only matter for the case of asymmetric outside options. We distinguish between the bargaining games under individual versus joint asset ownership.

Consider first the bargaining game under joint asset ownership with binding wealth constraints, where Alice wants to leave Bob to partner with Charles. Because the agreement of all three parties is required, any reasonable bargaining involves trilateral bargaining. While there may be many bargaining protocols that affect the distribution of rents between the three parties, the key insight is that the critical threshold $\widehat{\pi}_J(\sigma, w)$ from Lemma 2 does not depend on the specific distribution of these rents. This threshold only depends on the feasibility of obtaining an agreement between Alice, Bob and Charles that satisfies all three participation constraints. Specifically, at $\pi = \widehat{\pi}_J(\sigma, w)$ both Alice and Bob are indifferent between dissolving their partnership and staying together (each getting $U(\pi)$), while Charles receives the minimum equity stake $\gamma = 1 - \alpha^{\max}$. For any $\pi > \widehat{\pi}_J(\sigma, w)$ it is impossible to get a tripartite agreement, and for any $\pi \leq \widehat{\pi}_J(\sigma, w)$ it is always possible get such an agreement. As a consequence, the specific bargaining protocol actually does not matter for the partners' decision to stay together or to do a buyout.

Under individual asset ownership with binding wealth constraints we know from Lemma 1 that there exists a critical threshold $\widehat{\pi}_I(\sigma, w)$, such that Alice leaves Bob whenever $\pi < \widehat{\pi}_I(\sigma, w)$, and stays whenever $\pi \geq \widehat{\pi}_I(\sigma, w)$. Again we argue that reasonable alternative bargaining protocols may generate different utilities, but the critical threshold remains unaffected. One important restriction of the bargaining protocol by Hart and Mas-Colell (1996) is that at any point in time only one party can make an offer. Consider relaxing this assumption, and suppose that there can be simultaneous offers. In particular assume that the unique partner (Alice) can hold an auction for offers from the non-unique partners (Bob and Charles). Such an auction game results in a standard Bertrand pricing. It is easy to show that these Bertrand offers are more favorable to Alice than the bargaining outcome under the Hart and Mas-Colell protocol. However, since the auction is always won by the player with the highest valuation, it continues to be true that Alice teams up with Bob whenever $\pi \geq \widehat{\pi}_I(\sigma, w)$, and with Charles whenever $\pi < \widehat{\pi}_I(\sigma, w)$. Again we find that the critical threshold $\widehat{\pi}_I(\sigma, w)$ remains unaffected by the specific bargaining protocol.

Proof of Proposition 2.

Consider individual asset ownership. At date 1 partner $i = A, B$ chooses his specific investment r_i to maximize his expected utility:[2]

$$\begin{aligned} EU_I(r_i, r_j) &= p(r_i, r_j) \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_I(\pi_H, \sigma, w)\right] \\ &\quad + (1 - p(r_i, r_j)) \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_I(\pi_L, \sigma, w)\right] - \psi(r_i), \end{aligned}$$

where $j \in \{A, B\}$ and $j \neq i$. The equilibrium investment levels $r^*_{A(I)}(w)$ and $r^*_{B(I)}(w)$ under individual asset ownership are then characterized by the first-order conditions:

$$\frac{\partial p(r_A, r_B)}{\partial r_i} \Phi_I(w) = \psi'(r_i), \quad i = A, B,$$

where, using $V_I(\pi_L, \sigma, w) = U(\sigma)$,

$$\begin{aligned} \Phi_I(w) &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_I(\pi_H, \sigma, w)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) U(\sigma)\right]. \end{aligned}$$

Because Alice and Bob are symmetric at date 1, their investment levels $r^*_{A(I)}(w)$ and $r^*_{B(I)}(w)$ must be also symmetric in equilibrium. We define $r^*_I(w) \equiv r^*_{A(I)}(w) = r^*_{B(I)}(w)$ as the equilibrium relation-specific investment of a partner under individual asset ownership.

[2] Note that $U(\sigma) > U(\pi_L)$ when Alice and Bob each found an alternative partner; thus, $\max\{U(\pi_L), U(\sigma)\} = U(\sigma)$.

Likewise, the expected utility of partner $i = A, B$ at date 1 under joint asset ownership is given by

$$\begin{aligned} EU_J(p_i, p_j) &= p(r_i, r_j)\left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)V_J(\pi_H, \sigma, w)\right] \\ &\quad +(1-p(r_i, r_j))\left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_J(\pi_L, \sigma, w)\right] - \psi(r_i). \end{aligned}$$

The following first-order conditions define the equilibrium investment levels $r^*_{A(J)}(w)$ and $r^*_{B(J)}(w)$ under joint asset ownership:

$$\frac{\partial p(r_A, r_B)}{\partial r_i}\Phi_J(w) = \psi'(r_i) \quad i = A, B,$$

where, using $V_J(\pi_H, \sigma, w) = 2U(\pi_H)$,

$$\begin{aligned} \Phi_J(w) &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q)2U(\pi_H)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q)V_J(\pi_L, \sigma, w)\right]. \end{aligned}$$

Again, the Nash equilibrium is symmetric; we thus define $r^*_J(w) \equiv r^*_{A(J)}(w) = r^*_{B(J)}(w)$ as the equilibrium relation-specific investment of a partner under joint asset ownership.

Next, we define

$$\begin{aligned} F &\equiv \frac{\partial p(r_A, r_B)}{\partial r_A}\Phi_k(w) - \psi'(r_A) = 0 \\ G &\equiv \frac{\partial p(r_A, r_B)}{\partial r_B}\Phi_k(w) - \psi'(r_B) = 0, \end{aligned}$$

where $k \in \{I, J\}$. Applying Cramer's Rule we get

$$\frac{dr^*_{A(k)}(w)}{dw} = \frac{\det(X_1)}{\det(X_2)},$$

where

$$X_1 = \begin{pmatrix} -\frac{\partial F}{\partial w} & \frac{\partial F}{\partial r_B} \\ -\frac{\partial G}{\partial w} & \frac{\partial G}{\partial r_B} \end{pmatrix} \qquad X_2 = \begin{pmatrix} \frac{\partial F}{\partial r_A} & \frac{\partial F}{\partial r_B} \\ \frac{\partial G}{\partial r_A} & \frac{\partial G}{\partial r_B} \end{pmatrix}.$$

Because $U_i(\cdot)$, $i = A, B$, is concave, X_2 must be negative definite, so that $\det(X_2) > 0$. Thus, $dr^*_{A(k)}(w)/dw > 0$ if

$$\det(X_1) = -\frac{\partial F}{\partial w}\frac{\partial G}{\partial r_B} + \frac{\partial G}{\partial w}\frac{\partial F}{\partial r_B} > 0.$$

The second-order condition for $r^*_{B(k)}(w)$ implies $\partial G/\partial r_B < 0$. Moreover,

$$\frac{\partial F}{\partial r_B} = \frac{\partial^2 p(\cdot)}{\partial r_A \partial r_B}\Phi_k(w)$$

and

$$\frac{\partial F}{\partial w} = \frac{\partial p(\cdot)}{\partial r_A}\frac{d\Phi_k(w)}{dw} \qquad \frac{\partial G}{\partial w} = \frac{\partial p(\cdot)}{\partial r_B}\frac{d\Phi_k(w)}{dw},$$

where

$$\begin{aligned}\frac{d\Phi_I(w)}{dw} &= q(1-q)\frac{dV_I(w,\pi_H,\sigma)}{dw}\\ \frac{d\Phi_J(w)}{dw} &= -q(1-q)\frac{dV_J(w,\pi_L,\sigma)}{dw}.\end{aligned}$$

For individual asset ownership, recall that $dV_I(w,\pi_H,\sigma)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$, which implies that $\partial F/\partial w > 0$ and $\partial G/\partial w > 0$ for $\underline{w}_I \leq w < \overline{w}_I$. Thus, $dr^*_{A(I)}(w)/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$ and $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$, where κ is the lower bound of the cross-partial satisfying $\det(X_1) = 0$. Symmetry implies $dr^*_{A(I)}(w)/dw = dr^*_{B(I)}(w)/dw$. For joint asset ownership, recall that $dV_J(w,\pi_L,\sigma)/dw > 0$ for $\underline{w}_J \leq w < \overline{w}_J$, so that $\partial F/\partial w < 0$ and $\partial G/\partial w < 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Thus, $dr^*_{A(J)}(w)/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$ and $\partial^2 p(\cdot)/(\partial r_A \partial r_B) > -\kappa$. Due to symmetry, $dr^*_{A(J)}(w)/dw = dr^*_{B(J)}(w)/dw$.

For $w \geq \max\{\overline{w}_I, \overline{w}_J\}$ we know that $V_I(w,\pi_H,\sigma) = 2U(\pi_H)$ (individual ownership), and $V_J(w,\pi_L,\sigma) = U(\sigma)$ (joint ownership). Thus, we have $\Phi_I(w) = \Phi_J(w)$ for $w \geq \max\{\overline{w}_I, \overline{w}_J\}$, so that $r^*_I(w) = r^*_J(w)$. Furthermore, because $dr^*_I/dw > 0$ for $\underline{w}_I \leq w < \overline{w}_I$, and $dr^*_J/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$, we can infer that $r^*_J(w) > r^*_I(w)$ for $w < \max\{\overline{w}_I, \overline{w}_J\}$. □

Proof of Lemma 3.

Under individual asset ownership the expected utility of Alice at date 0 is given by

$$\begin{aligned}EU^A_I(p^*,w) &= p^*\left[q^2\max\{U(\pi_H),U(\sigma)\} + (1-q)^2U(\pi_H) + q(1-q)V_I(\pi_H,\sigma,w)\right]\\ &\quad +(1-p^*)\left[q^2U(\sigma) + (1-q)^2U(\pi_L) + q(1-q)V_I(\pi_L,\sigma,w)\right] - \psi(r^*_{A(I)}),\end{aligned}$$

with $p^* \equiv p(r^*_{A(I)}, r^*_{B(I)})$ and $V_I(\pi_L, \sigma, w) = U(\sigma)$. The expected utility of Bob is symmetric. Applying the Envelope Theorem we get

$$\frac{dEU_I^A(p^*, w)}{dw} = \frac{\partial EU_I^A(p^*, w)}{\partial r_{B(I)}} \frac{dr^*_{B(I)}}{dw} + p^* q(1-q) \frac{\partial V_I(\pi_H, \sigma, w)}{\partial w}.$$

Note that $\partial EU_I^A(\cdot)/\partial r_{B(I)} > 0$. We need to consider three cases: (i) $w \leq \underline{w}_I$, (ii) $w > \overline{w}_I$; and (iii), $\underline{w}_I < w \leq \overline{w}_I$. For the first two cases we know that $dr^*_{B(I)}/dw = 0$ and $\partial V_I/\partial w = 0$; thus, $dEU_I^A(p^*, w)/dw = 0$. For $\underline{w}_I < w \leq \overline{w}_I$ we know that $dr^*_{B(I)}/dw > 0$ and $\partial V_I/\partial w > 0$; thus, $dEU_I^A(p^*, w)/dw > 0$. This also implies that $EU_I^A(p^*, w)$ is maximized for $w \geq \overline{w}_I$. □

Proof of Lemma 4.

Under joint asset ownership the expected utility of Alice at date 0 is given by

$$\begin{aligned} EU_J^A(p^*, w) &= p^* \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) V_J(\pi_H, \sigma, w)\right] \\ &\quad + (1-p^*) \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_J(\pi_L, \sigma, w)\right] - \psi(r^*_{A(I)}), \end{aligned}$$

with $p^* = p(r^*_{A(J)}, r^*_{B(J)})$ and $V_J(\pi_H, \sigma, w) = 2U(\pi_H)$. The expected utility of Bob is symmetric. Applying the Envelope Theorem yields

$$\begin{aligned} \frac{dEU_J^A(p^*, w)}{dw} &= \frac{\partial EU_J^A(p^*, w)}{\partial r_{B(J)}} \frac{dr^*_{B(J)}}{dw} + (1-p^*) q(1-q) \frac{\partial V_J(\pi_L, \sigma, w)}{\partial w} \\ &= \underbrace{\Phi_J(w) \frac{\partial p(\cdot)}{\partial r_{B(J)}} \frac{dr^*_{B(J)}}{dw}}_{\equiv \psi_1} + \underbrace{(1-p^*) q(1-q) \frac{\partial V_J(\pi_L, \sigma, w)}{\partial w}}_{\equiv \psi_2}, \end{aligned}$$

where

$$\begin{aligned} \Phi_J(w) &= \left[q^2 \max\{U(\pi_H), U(\sigma)\} + (1-q)^2 U(\pi_H) + q(1-q) 2U(\pi_H)\right] \\ &\quad - \left[q^2 U(\sigma) + (1-q)^2 U(\pi_L) + q(1-q) V_J(\pi_L, \sigma, w)\right] > 0. \end{aligned}$$

By definition, $\partial p(\cdot)/\partial r_{B(J)} > 0$. Moreover, recall from Proposition 2 that $dr^*_{B(J)}/dw < 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Thus, $\psi_1 < 0$ for $\underline{w}_J \leq w < \overline{w}_J$. Furthermore, $\partial V_J(\pi_L, \sigma, w)/\partial w > 0$ for $\underline{w}_J \leq w < \overline{w}_J$, so that $\psi_2 > 0$ for $\underline{w}_J \leq w < \overline{w}_J$. We define w^*_J as the wealth level which satisfies $dEU_J^A(p^*, w)/dw = 0$ for $\underline{w}_J \leq w < \overline{w}_J$, and thus maximizes Alice's expected utility at date 0. Note that $\underline{w}_J \leq w_J^* < \overline{w}_J$ because $dr^*_{B(J)}/dw = 0$ and $\partial V_J(\pi_L, \sigma, w)/\partial w = 0$ for $w < \underline{w}_J$ and $w \geq \overline{w}_J$. To summarize, (i) $dEU_J^A(\cdot)/dw = 0$ for $w \leq \underline{w}_J$, $w \geq \overline{w}_J$, and $w = w_J^*$ (as $\psi_1 + \psi_2 = 0$), (ii) $dEU_J^A(\cdot)/dw > 0$ for $\underline{w}_J < w < w_J^*$ (as $\psi_1 + \psi_2 > 0$); and (iii), $dEU_J^A(\cdot)/dw < 0$ for $w_J^* < w < \overline{w}_J$ (as $\psi_1 + \psi_2 < 0$).

Finally note that $\lim_{\pi_H \to \infty} \Phi_J(w) = \infty$ as $dU(\pi_H)/d\pi_H > 0$ with $\lim_{\pi_H \to \infty} U(\pi_H) = \infty$. This implies that $\lim_{\pi_H \to \infty} \psi_1 = -\infty$ for $\underline{w}_J \leq w < \overline{w}_J$, while $\sup(\psi_2) < \infty$. Thus, there exists a threshold $\widehat{\pi}_H$ such that $dEU_J^A(\cdot)/dw < 0$ for all $\pi_H \geq \widehat{\pi}_H$ and $w \in (\underline{w}_J, \overline{w}_J)$, which implies a corner solution with $w_J^* \leq \underline{w}_J$. □

Proof of Proposition 3.

Suppose $w \geq \overline{w} = \max\{\overline{w}_I, \overline{w}_J\}$. Under individual asset ownership, $V_I(\pi_H, \sigma, w) = 2U(\pi_H)$ for $w \geq \overline{w}_I$. Under joint asset ownership, $V_J(\pi_L, \sigma, w) = V_I(\pi_L, \sigma, w) = U(\widehat{\alpha}_I; \sigma)$ for $w \geq \overline{w}_J$. Moreover, recall from Proposition 2 that $r_I^*(w) = r_J^*(w)$ for all $w \geq \overline{w}$. Thus, $EU_I(r_I^*, w) = EU_J(r_J^*, w)$ for $w \geq \overline{w}$.

Next, recall from Lemma 3 that $dEU_I(\cdot)/dw > 0$ for $\underline{w}_I < w \leq \overline{w}_I$, where $EU_I(\cdot)$ is maximized for $w \geq \overline{w}_I$. Moreover, we know from Lemma 4 that $dEU_J(\cdot)/dw > 0$ for $\underline{w}_J < w < w_J^*$, and $dEU_J(\cdot)/dw < 0$ for $w_J^* < w \leq \overline{w}_J$, where $EU_J(\cdot)$ is maximized when $w = w_J^*$. This implies that $EU_J(\cdot) > EU_I(\cdot)$ for $w \in [w_J^*, \overline{w})$.

Finally we examine whether $EU_I(\cdot) > EU_J(\cdot)$ for some $w < w_J^*$. Suppose $\pi_H \to \pi_L$. We can then immediately see that $r_I^*(w) = r_J^*(w) = 0$, and hence, $EU_I(\cdot) > EU_J(\cdot)$. We define w_0 as the critical wealth level so that $EU_J(\cdot) > EU_I(\cdot)$ for $w \in [w_0, \overline{w})$. Note that $w_0 < w_J^*$ because $EU_J(\cdot) > EU_I(\cdot)$ for $w_J^* \leq w < \overline{w}$. Moreover, $w_0 \geq 0$ because, when π_H is sufficiently high, $EU_J(\cdot) > EU_I(\cdot)$ even for $w = 0$. Thus, joint asset ownership is strictly optimal for $w_0 \leq w < \overline{w}$, with $w_0 \in [0, w_J^*]$. According to Lemma 4, the optimal wealth level is then $w_J^* \in [0, \overline{w})$, with $w_J^* \leq \underline{w}_J$ for all $\pi_H \geq \widehat{\pi}_H$. □

Outside Financing.

Consider date 2 and assume w.l.o.g. that only Alice found an alternative partner, Charles. Suppose the partners' wealth constraints are binding, i.e., they cannot fully eliminate the ex-post inefficiencies associated with asymmetric outside options. We now ask whether raising the amount $K > 0$ from an outside investor for (additional) transfer payments, can lead to a Pareto improvement (which is required for changing the renegotiation outcome).

Assume a competitive capital market, with cost of capital $r \geq 0$. Moreover, recall that the only payoff in our model is y, which is realized at date 4. Thus, the partners can only offer the outside investor, who does not contribute to the production process, a share δ on the return y in exchange for K, with expected values $\pi = \int_{\underline{y}}^{\overline{y}} y d\Omega_{in}(y)$ (inside prospect) and $\sigma = \int y d\Omega_{out}(y)$ (outside prospect).

Consider joint asset ownership. With insufficient wealth ($w < \overline{w}_J$), Alice would need to offer Bob the stake $\widehat{\beta}_J$ in the new partnership with Charles – in addition to the transfer w – to buy out her asset (note that $\widehat{\beta}_J$ already accounts for the transfer w, i.e., $\widehat{\beta}_J = \widehat{\beta}_J(w)$). This, however, compromises effort incentives for Alice and Charles. Alice can also raise the amount $K \in [0, \overline{w}_J - w]$ to mitigate (or even to eliminate) the inefficiency associated with the buy out. Alice can then offer Bob the payment K and the new stake $\widehat{\beta}_J(K) \in [0, \widehat{\beta}_J]$, with $d\widehat{\beta}_J(K)/dK < 0$ and $\widehat{\beta}_J(K = \overline{w}_J - w) = 0$. Bob accepts the amount K in exchange for a lower equity stake $\widehat{\beta}_J(K)$ when

$$K = \left[\widehat{\beta}_J - \widehat{\beta}_J(K)\right] \mu \left(e_A(K) e_C(K)\right) \sigma. \tag{A.20}$$

Furthermore, the zero profit condition for the outside investor, who gets the stake $\widehat{\delta}_J(K)$, implies

$$\widehat{\delta}_J(K) \mu \left(e_A(K) e_C(K)\right) \sigma = (1 + r) K.$$

Combining the two conditions we get

$$\widehat{\delta}_J(K) = (1 + r) \left[\widehat{\beta}_J - \widehat{\beta}_J(K)\right].$$

Thus, for $r \geq 0$, the equity stake $\widehat{\delta}_J(K)$ for the (unproductive) outside investor is at least as high as the equity stake that Bob relinquishes in exchange for the extra transfer K. This implies that $d\widehat{\alpha}_J/dK \leq 0$ and $d\widehat{\gamma}_J/dK \leq 0$, and therefore, $de_A(K)/dK \leq 0$ and $de_C(K)/dK \leq 0$.

For any transfer $K \in [0, \overline{w}_J - w]$, the expected joint utility for Alice and Bob under joint asset ownership is

$$U_A(K) + U_B(K) = \widehat{\alpha}_J(K) \mu \left(e_A(K) e_C(K)\right) \sigma - c\left(e_A(K)\right) - w + \widehat{\beta}_J(K) \mu \left(e_A(K) e_C(K)\right) \sigma + K + w.$$

Using (A.20) we can write this as

$$U_A(K) + U_B(K) = \widehat{\alpha}_J(K) \mu \left(e_A(K) e_C(K)\right) \sigma - c\left(e_A(K)\right) + \widehat{\beta}_J \mu \left(e_A(K) e_C(K)\right) \sigma.$$

Note that Alice chooses $e_A(K)$ such that $dU_A(K)/de_A = 0$; thus,

$$\begin{aligned}\frac{d}{dK}\left[U_A(K)+U_B(K)\right] &= \overbrace{\frac{d\widehat{\alpha}_J(K)}{dK}}^{\leq 0}\mu\left(e_A(K)e_C(K)\right)\sigma \\ &\quad +\widehat{\alpha}_J(K)\mu'\left(e_A(K)e_C(K)\right)e_A(K)\overbrace{\frac{de_C(K)}{dK}}^{\leq 0}\sigma \\ &\quad +\widehat{\beta}_J\mu'\left(e_A(K)e_C(K)\right)\sigma\left[\underbrace{\frac{de_A(K)}{dK}}_{\leq 0}e_C(K)+e_A(K)\underbrace{\frac{de_C(K)}{dK}}_{\leq 0}\right].\end{aligned}$$

Consequently, $d\left[U_A(K)+U_B(K)\right]/dK \leq 0$, which implies that raising $K > 0$ for (additional) transfer payments is not Pareto improving under joint ownership. Hence, $K_J^* = 0$.

Now consider individual asset ownership, and suppose that Alice decided to stay but renegotiated a more favorable profit share. With insufficient wealth ($w < \overline{w}_I$), Bob can buy back some of his original profit share by paying Alice the amount w, but this is not enough to fully eliminate the ex-post inefficiency, as in equilibrium we still have $\alpha_I^* > \beta_I^*$ (note that α_I^* and β_I^* already account for the transfer w, i.e., $\alpha_I^* = \alpha_I^*(w)$ and $\beta_I^* = \beta_I^*(w)$). Bob can also raise the amount $K \in [0, \overline{w}_I - w]$ to buy back more profit shares, in order to better align team incentives. The new payoffs for Alice and Bob are then given by $(1-\delta_I^*(K))\alpha_I^*(K)\pi$ and $(1-\delta_I^*(K))\beta_I^*(K)\pi$, respectively, where $\delta_I^*(K)$ is the investor's profit share. Alice accepts the transfer K (in addition to the transfer w) in exchange for relinquishing some of her profit shares when

$$K = \alpha_I^*\mu(e_Ae_B)\pi - c_A(e_A) - \left[\widetilde{\alpha}_I(K)\mu(e_A(K)e_B(K))\pi - c_A(e_A(K))\right], \tag{A.21}$$

where $\widetilde{\alpha}_I(K) = (1-\delta_I^*(K))\alpha_I^*(K)$ is Alice's net profit share. Because $d\widetilde{\alpha}_I(K)/dK < 0$, it is straightforward to show that $de_A(K)/dK < 0$. Moreover, the outside investor's stake $\delta_I^*(K)$ is defined by his zero profit condition:

$$\delta_I^*(K)\mu(e_A(K)e_B(K))\pi = (1+r)K.$$

For any transfer $K \in [0, \overline{w}_I - w]$, the expected joint utility under individual asset ownership with Alice staying with Bob, is

$$\begin{aligned} U_A(K) + U_B(K) &= \widetilde{\alpha}_I(K)\mu(e_A(K)e_B(K))\pi - c_A(e_A(K)) + w + K \\ &\quad + \widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)) - w, \end{aligned}$$

where $\widetilde{\beta}_I(K) = (1 - \delta_I^*(K))\beta_I^*(K)$ is Bob's net profit share. Using (A.21) we can write the expected joint utility as

$$U_A(K) + U_B(K) = \alpha_I^*\mu(e_A e_B)\pi - c_A(e_A) + \widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)).$$

Note that $dU_A(K)/dK = 0$. Moreover, the total surplus that is split between Alice and Bob, $(1 - \delta_I^*(K))\pi$, is decreasing in K, where the profit share $\delta_I^*(K)\pi$ goes to an unproductive party (the investor). Thus, $d[U_A + U_B(K)]/dK < 0$. Consequently, raising $K > 0$ for (additional) transfer payments is not Pareto improving under individual asset ownership (case of staying and renegotiation), so that $K_I^* = 0$.

Now consider the case where Alice wants to leave Bob under individual asset ownership. Bob can then offer Alice a lump sum payment (equal to w) and a higher profit share to make her stay. Without sufficient wealth ($w < \overline{w}_I$), Bob's retention offer either is not enough to convince Alice to stay, or ensures that Alice stays but with her getting more than half of the surplus. Bob can also raise the amount $K \in [0, \overline{w}_I - w]$ to make Alice a more efficient retention offer.

First suppose that Bob's original offer (with $K = 0$) is enough to retain Alice, but with an unequal split of surplus, so that $\alpha_I^* > \beta_I^*$. Bob can then raise the amount $K > 0$ to buy additional profit shares from Alice, in order to better align team incentives. Alice's new payoff is then given by $\widetilde{\alpha}_I(K)\pi$, and Bob's by $\widetilde{\beta}_I(K)\pi$, where $\widetilde{\alpha}_I = (1 - \delta_I^*(K))\alpha_I^*(K)$ and $\widetilde{\beta}_I(K) = (1 - \delta_I^*(K))\beta_I^*(K)$. Alice accepts Bob's retention offer when

$$K = \widehat{\alpha}_I\mu(e_A e_C)\sigma - c_A(e_A) - \left[\widetilde{\alpha}_I(K)\mu(e_A(K)e_B(K))\pi - c_A(e_A(K))\right]. \tag{A.22}$$

Again we have $de_A(K)/dK < 0$ because $d\widetilde{\alpha}_I(K)/dK < 0$. Furthermore, the outside investor's stake $\delta_I^*(K)$ is defined by

$$\delta_I^*(K)\mu(e_A(K)e_B(K))\pi = (1 + r)K.$$

For any transfer $K \in [0, \overline{w}_I - w]$, the expected joint utility for Alice and Bob under individual asset ownership – with Bob's original retention offer ($K = 0$) being enough to keep Alice – is given by

$$U_A(K) + U_B(K) = \widetilde{\alpha}_I(K)\mu(e_A(K)e_B(K))\pi - c_A(e_A(K)) + w + K$$
$$+\widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)) - w.$$

Using (A.22) we can write the expected joint utility as

$$U_A(K) + U_B(K) = \widehat{\alpha}_I\mu(e_Ae_C)\sigma - c_A(e_A) + \widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)).$$

Again we note that $dU_A(K)/dK = 0$. In addition, the total surplus that is split between the two productive partners, $(1 - \delta_I^*(K))\pi$, is decreasing in K, where the profit share $\delta_I^*(K)$ goes to the (unproductive) outside investor. Thus, $d[U_A + U_B(K)]/dK < 0$. Consequently, $K_I^* = 0$ in case Bob's original retention offer ($K = 0$) can only retain Alice with an unbalanced split of surplus.

Next consider the case where Bob's original offer ($K = 0$) is not enough to retain Alice (in which case Bob's expected utility is zero). Raising $K > 0$ is then optimal for Bob when

$$U_B(K) = \widetilde{\beta}_I(K)\mu(e_A(K)e_B(K))\pi - c_B(e_B(K)) - w > 0.$$

Note that $U_B(0) = 0$ (in this case Bob cannot retain Alice, so he does not offer her the lump sum payment w). Moreover, from the above we know that $dU_A(K)/dK = 0$ and $d[U_A + U_B(K)]/dK < 0$, which implies that $dU_B(K)/dK < 0$. Thus, this condition is not satisfied for any $K > 0$, so that $K_I^* = 0$ in case Bob's original retention offer ($K = 0$) is insufficient to retain Alice.

The fundamental reason why outside investors cannot improve efficiency is that their return on investment must come from the profits of the venture. Giving the investors a share on those profits creates an inefficiency that is at least as large as the inefficiency that their investments are supposed to solve. It is worth noting that in our base model the venture does not generate any risk-free returns. A transfer of safe returns would not create those inefficiencies as they do not affect team incentives. It is possible to extend our base model to allow for some risk-free returns. This can be in the form of safe (interim or final) profits, or any fixed asset liquidation values. The main insight from such an extension is that we can add the risk-free returns to our measure of partner wealth. Specifically we can redefine a partner's effective wealth as the sum of exogenous wealth plus half of his/her share on the risk-free returns of the venture. In a renegotiation there are two ways that partners can use their respective shares of the risk-free returns. Suppose Bob wants to make a transfer to Alice. He can simply assign a part or all of his share on the safe returns to Alice, directly giving her a first claim on the safe returns. Alternatively, the two partners can raise some safe debt from an outside investor, and use the proceeds to pay Alice. In the latter case, outside investors can play a role, but again they cannot improve the outcomes that can be achieved by the two partners alone. In particular, the two partners prefer to internally reassign the safe returns without the help of an outside investor whenever there is a cost of raising outside funds (e.g. when $r > 0$).

Preferences – Ex-ante Asymmetric Outside Options.

W.l.o.g. we focus on Alice's preference for the allocation of control rights; Bob's preference is symmetric. Let $\overline{q}_i \equiv 1 - q_i$, $i = A, B$. Alice's expected utility at date 0 under joint asset ownership is

$$U_A^J = q_A q_B \frac{\sigma}{2} + (\overline{q}_A q_B + q_A \overline{q}_B + \overline{q}_A \overline{q}_B) \frac{\pi}{2}.$$

Likewise, Alice's expected utility at date 0 under individual asset ownership is

$$U_A^I = q_A q_B \frac{\sigma}{2} + q_A \overline{q}_B \frac{\sigma}{2} + \overline{q}_A \overline{q}_B \frac{\pi}{2}.$$

Alice prefers joint asset ownership if $U_A^J > U_A^I$, which is equivalent to

$$q_A < \widehat{q}_A = \left[\left(\frac{1 - q_B}{q_B} \right) \left(\frac{\sigma - \pi}{\pi} \right) + 1 \right]^{-1}.$$

Moreover, after some simplifications we find that

$$\begin{aligned} \frac{d\widehat{q}_A}{dq_B} &= \left[(1 - q_B) \left(\frac{\sigma - \pi}{\pi} \right) + q_B \right]^{-2} \left(\frac{\sigma - \pi}{\pi} \right) > 0 \\ \frac{d^2\widehat{q}_A}{dq_B^2} &= -2 \left[(1 - q_B) \left(\frac{\sigma - \pi}{\pi} \right) + q_B \right]^{-3} \left(\frac{\sigma - \pi}{\pi^2} \right) (2\pi - \sigma). \end{aligned}$$

Note that $d^2\widehat{q}_A/dq_B^2 < 0$ if $\pi > \sigma/2$, and $d^2\widehat{q}_A/dq_B^2 > 0$ if $\pi < \sigma/2$.